Holes

Example 1:

Hole(s):

Limit Notation:

$$\lim_{x \to 3^-} f(x) =$$ $$\lim_{x \to 3^+} f(x) =$$

As x approaches 3 from the left the $f(x)$…

As x approaches 3 from the right the $f(x)$…

Example 2:

$$g(x) = \frac{10x + 30}{x^2 + x - 6}$$

Limit Notation of Hole(s):
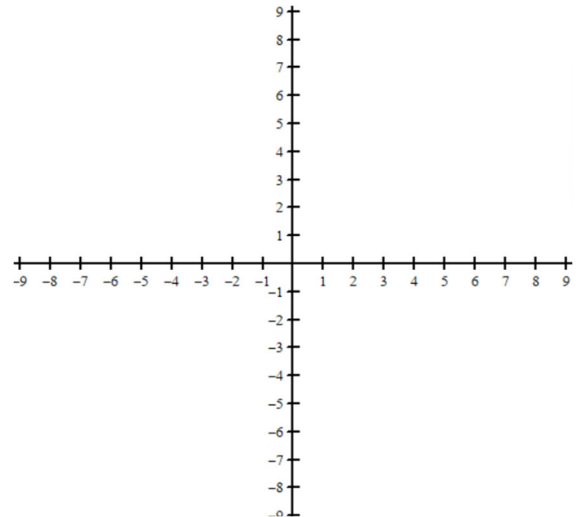
Numerically

As x approaches -3 from the left the $g(x)$…

x	$g(x)$
-3.1	
-3.01	
-3.001	
-3.0001	

As x approaches -3 from the right the $g(x)$…

x	$g(x)$
-2.9	
-2.99	
-2.999	
-2.9999	

Example 3:   **Can look like polynomials!**

$$g(x) = \frac{(-x^2 + 2x + 4)(x - 1)}{x - 1}$$
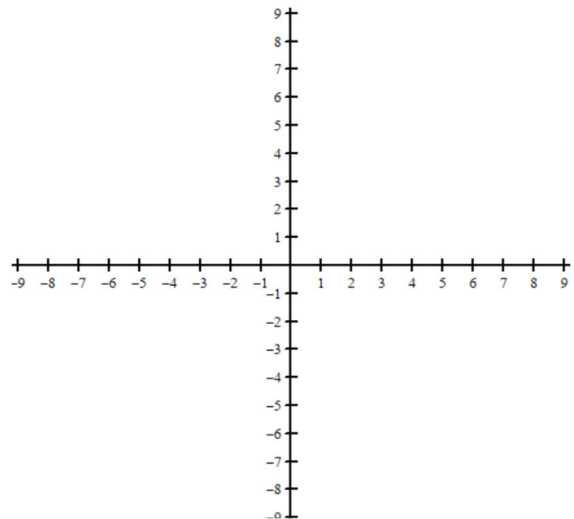
Holes

Let f be the rational function $f(x) = \frac{N(x)}{D(x)}$. The holes of the rational function occur at all common factors whose multiplicity in the numerator is greater than or equal to the the multiplicity in the denominator.

Multiplicity

Example 4:

$$h(x) = \frac{(x - 2)^2}{x^2 + 4x - 12}$$

Domain: Vertical Asymptote(s):

Hole(s):   Horizontal Asymptote:

Zero(s):   End Behavior:

1.10 Rational Functions and Holes

AP Precalculus

Find the hole(s) of the following rational function if one exists.

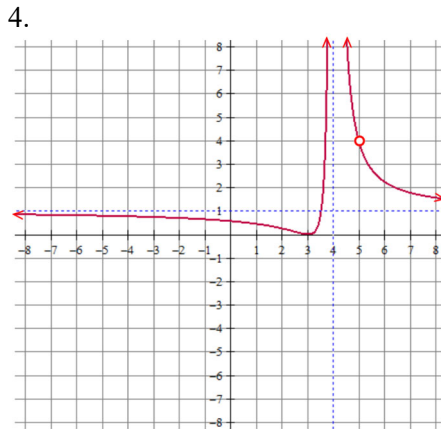1. $f(x) = \dfrac{x^2+3x}{x^2-9}$

Hole(s):

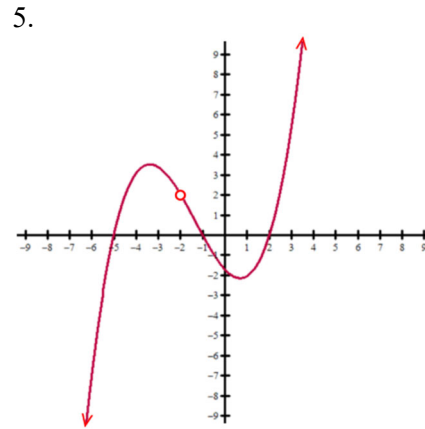2. $d(t) = \dfrac{t^2+t-20}{t+5}$
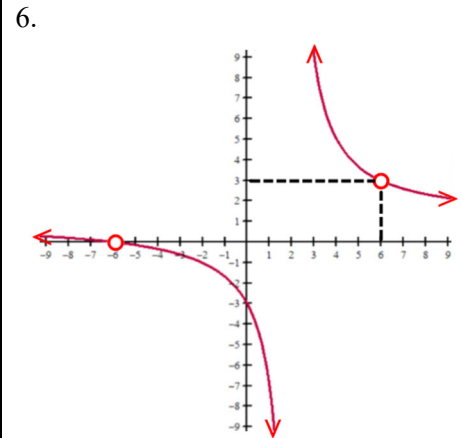
Hole(s):

3. $h(x) = \dfrac{(x+5)^2}{x^2+8x+15}$

Hole(s):

State the hole(s) of the following rational functions. Use limit notation.

4.

Limit Notation Hole(s):

5.

Limit Notation Hole(s):

6.

Limit Notation Hole(s):

CALCULATOR ACTIVE Complete the table to answer the following.

7. $f(x) = \dfrac{x^2-16}{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$							

Hole: Limit Notation of Hole:

CALCULATOR ACTIVE Complete the table to answer the following.

8. $f(x) = \dfrac{x^2 - 2x - 8}{4x + 8}$

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$							

Hole: Limit Notation of Hole:

Use the table of the rational function d to find the following.

9.

t	$d(t)$
-3.1	4.134
-3.01	4.15
-3.001	4.1893
-3.0001	4.1998
-3	undefined
-2.9999	4.2014
-2.999	4.231
-2.99	4.305
-2.9	4.37

a. Find $\lim\limits_{t \to -3^-} d(t) =$

b. Find $\lim\limits_{t \to -3^+} d(t) =$

c. As t approaches negative three from the left the $d(t)$...

d. As t approaches negative three from the right the $d(t)$...

Use the graph of the rational function f to find the following.

10.

a. $f(6) =$

b. $f(2) =$

c. $\lim\limits_{x \to -5^-} f(x) =$

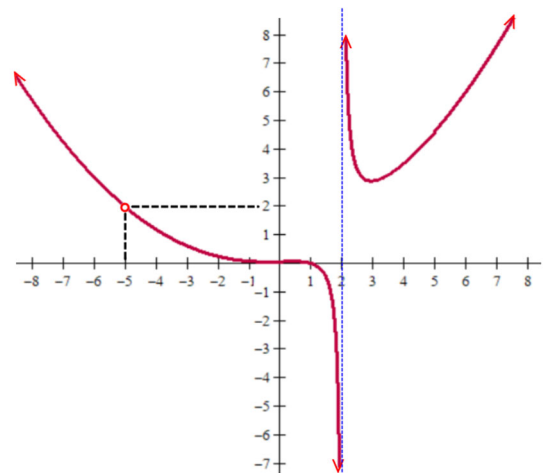d. $\lim\limits_{x \to -5^+} f(x) =$

e. $\lim\limits_{x \to 2^-} f(x) =$

f. $\lim\limits_{x \to 2^+} f(x) =$

g. $\lim\limits_{x \to -\infty} f(x) =$

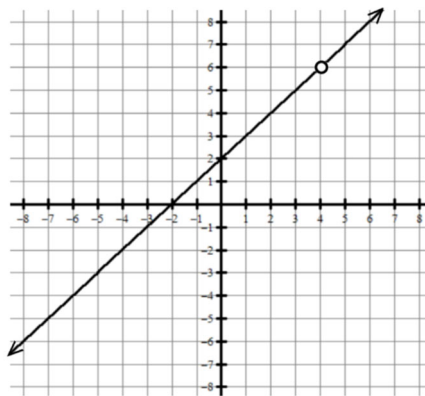h. $\lim\limits_{x \to \infty} f(x) =$

i. Domain $=$

Multiple Choice

11. The function f is given by $f(x) = \dfrac{x^2-9}{x^2+8x+15}$. Which of the following describes the function f ?

    (A) There is a hole at $x = 5$.

    (B) There is a hole at $x = -5$.

    (C) There is a hole at $x = 3$.

    (D) There is a hole at $x = -3$.

For questions 12-13 use the graph of f.

graph of f

12. The figure shows the graph of a function f. Which of the following could be an expression for the $f(x)$?

    (A) $\dfrac{(x+2)(x-4)}{(x+2)}$

    (B) $\dfrac{(x-2)(x+4)}{(x-2)}$

    (C) $\dfrac{(x+2)(x-4)}{(x-4)}$

    (D) $\dfrac{(x-2)(x+4)}{(x+4)}$

13. The figure shows the graph of a function f. Which of the following must be true?

    (A) $f(2) = 0$

    (B) $\lim\limits_{x \to 4^+} f(x) = \infty$

    (C) $\lim\limits_{x \to 4^+} f(x) = 3f(0)$

    (D) $f(-2) + f(0) = 0$