

# 1.10 Rational Functions and Holes

AP Precalculus

# 1.10 Practice

Find the hole(s) of the following rational function if one exists.

1.  $f(x) = \frac{x^2+3x}{x^2-9}$   
 ~~$x(x+3)$~~   
 ~~$(x+3)(x-3)$~~

Hole(s):  
 $x = -3$

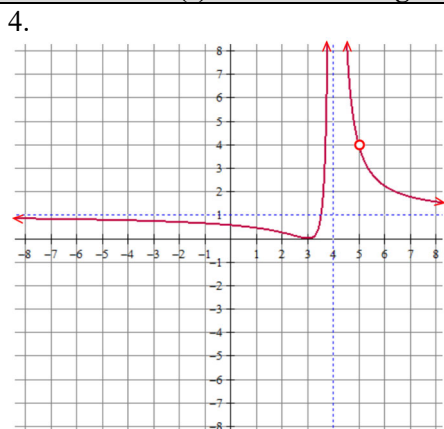
2.  $d(t) = \frac{t^2+t-20}{t+5}$   
 ~~$(t+5)(t-4)$~~

Hole(s):  
 $t = -5$

3.  $h(x) = \frac{(x+5)^2}{x^2+8x+15}$   
 ~~$(x+5)(x+5)$~~   
 ~~$(x+3)(x+5)$~~

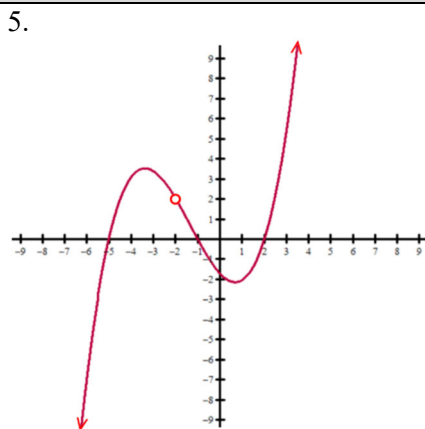
Hole(s):  
 $x = -5$

State the hole(s) of the following rational functions. Use limit notation.



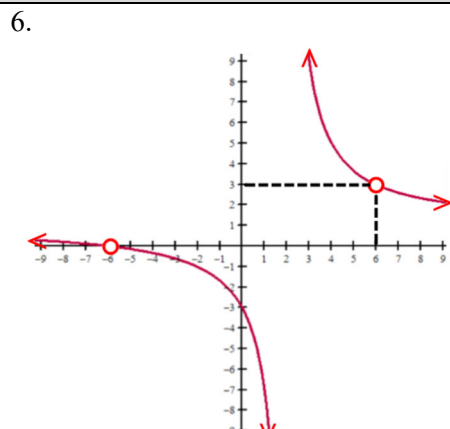
Limit Notation Hole(s):

$\lim_{x \rightarrow 5^-} f(x) = 4$      $\lim_{x \rightarrow 5^+} f(x) = 4$



Limit Notation Hole(s):

$\lim_{x \rightarrow -2^-} f(x) = 2$      $\lim_{x \rightarrow -2^+} f(x) = 2$



Limit Notation Hole(s):

$\lim_{x \rightarrow -6^-} f(x) = 0$      $\lim_{x \rightarrow -6^+} f(x) = 0$   
 $\lim_{x \rightarrow 6^-} f(x) = 3$      $\lim_{x \rightarrow 6^+} f(x) = 3$

**CALCULATOR ACTIVE** Complete the table to answer the following.

7.  $f(x) = \frac{x^2-16}{x-4}$

$x$	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	7.9	7.99	7.999	undefined	8.001	8.01	8.1

Hole:

$x = 4$

Limit Notation of Hole:

$\lim_{x \rightarrow 4^-} f(x) = 8$      $\lim_{x \rightarrow 4^+} f(x) = 8$

**CALCULATOR ACTIVE** Complete the table to answer the following.

8.  $f(x) = \frac{x^2 - 2x - 8}{4x + 8}$

$x$	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$	-1.525	-1.503	-1.50025	undefined	-1.49975	-1.498	-1.02

Hole:

Limit Notation of Hole:

$x = -2$

$\lim_{x \rightarrow -2^-} f(x) = -1.5$      $\lim_{x \rightarrow -2^+} f(x) = -1.5$

**Use the table of the rational function  $d$  to find the following.**

9.

$t$	$d(t)$
-3.1	4.134
-3.01	4.15
-3.001	4.1893
-3.0001	4.1998
-3	undefined
-2.9999	4.2014
-2.999	4.231
-2.99	4.305
-2.9	4.37

a. Find  $\lim_{t \rightarrow -3^-} d(t) = 4.2$

b. Find  $\lim_{t \rightarrow -3^+} d(t) = 4.2$

c. As  $t$  approaches negative three from the left the  $d(t)$ ...

d. As  $t$  approaches negative three from the right the  $d(t)$ ...

approaches 4.2

approaches 4.2

**Use the graph of the rational function  $f$  to find the following.**

10.

a.  $f(6) = 6$

b.  $f(2) = \text{DNE or undefined}$

c.  $\lim_{x \rightarrow -5^-} f(x) = 2$

d.  $\lim_{x \rightarrow -5^+} f(x) = 2$

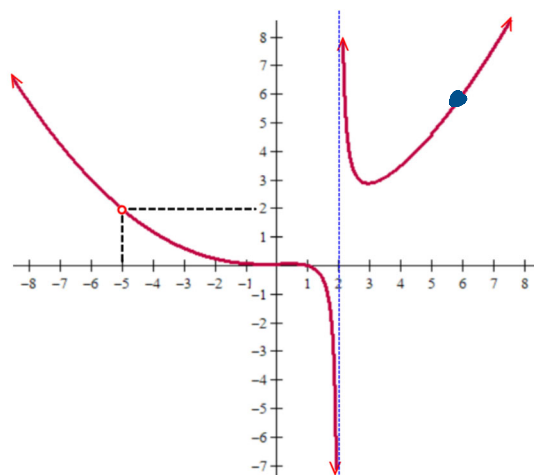
e.  $\lim_{x \rightarrow 2^-} f(x) = -\infty$

f.  $\lim_{x \rightarrow 2^+} f(x) = \infty$

g.  $\lim_{x \rightarrow -\infty} f(x) = \infty$

h.  $\lim_{x \rightarrow \infty} f(x) = \infty$

i. Domain =  $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$



Multiple Choice

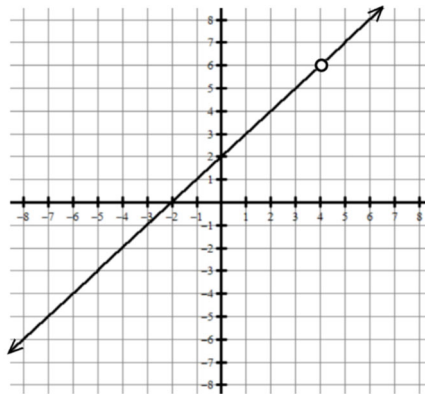
~~(x+3)~~(x-3)

11. The function  $f$  is given by  $f(x) = \frac{x^2-9}{x^2+8x+15}$ . Which of the following describes the function  $f$  ?

(x+5)~~(x+3)~~

- (A) There is a hole at  $x = 5$ .
- (B) There is a hole at  $x = -5$ .
- (C) There is a hole at  $x = 3$ .
- (D) There is a hole at  $x = -3$ .

For questions 12-13 use the graph of  $f$ .



graph of  $f$

12. The figure shows the graph of a function  $f$ . Which of the following could be an expression for the  $f(x)$  ?

(A)  $\frac{(x+2)(x-4)}{(x+2)}$

$x = -2$  is a zero so the factor  $(x + 2)$  must be in the numerator.

(B)  $\frac{(x-2)(x+4)}{(x-2)}$

There is a hole at  $x = 4$  so the factor  $(x - 4)$  must be in the numerator and denominator.

(C)  $\frac{(x+2)(x-4)}{(x-4)}$

(D)  $\frac{(x-2)(x+4)}{(x+4)}$

13. The figure shows the graph of a function  $f$ . Which of the following must be true?

(A)  $f(2) = 0$  *nope  $f(2) = 4$*

(B)  $\lim_{x \rightarrow 4^+} f(x) = \infty$  *nope*

(C)  $\lim_{x \rightarrow 4^+} f(x) = 3f(0)$  *YES! both = 6*

*$f(0) = 2$   
 $3 \cdot f(0)$   
 $3 \cdot 2$   
 $6$*

*$\lim_{x \rightarrow 4^+} f(x) = 6$*

(D)  $f(-2) + f(0) = 0$  *nope*  
 *$0 + 2 = 2$*