

# 1.11B Polynomial Long Division and Slant Asymptotes

## 1.11B Practice

AP Precalculus

Divide the following using long division or synthetic division.

1.  $\frac{3x^3 - 4x^2 - 3}{x^2 + 5x + 1}$

$$\begin{array}{r} 3x - 19 \\ x^2 + 5x + 1 \overline{) 3x^3 - 4x^2 + 0x - 3} \\ \underline{-(3x^3 + 15x^2 + 3x)} \phantom{-3} \\ -19x^2 - 3x - 3 \\ \underline{-(-19x^2 - 95x - 19)} \\ 92x + 16 \end{array}$$

$$3x - 19 + \frac{92x + 16}{x^2 + 5x + 1}$$

2.  $\frac{x^3 - 4x^2 + 6x - 4}{x - 2}$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 6 & -4 \\ & & 2 & -4 & 4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

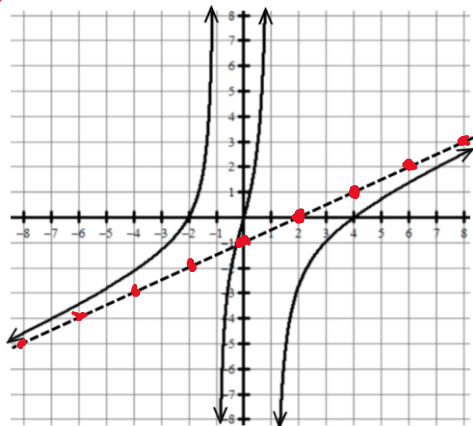
Remainder

$$x^2 - 2x + 2 + \frac{0}{x - 2}$$

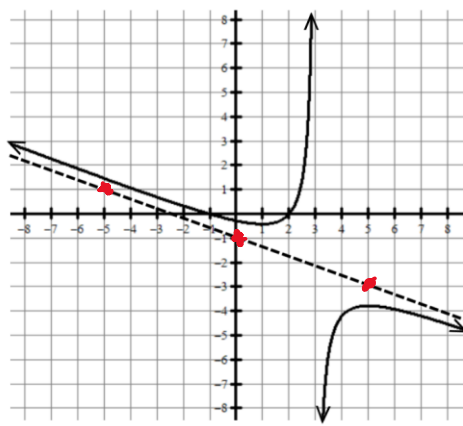
$$x^2 - 2x + 2$$

Use the graph of  $f$  to write the equation of the slant asymptote.

3.  $y = \frac{1}{2}x - 1$



4.  $y = -\frac{2}{5}x - 1$



Determine if the following functions have a horizontal asymptote, slant asymptote, or neither.

5.  $f(x) = \frac{4x^5 - 3x^3 + 4x + 1}{5x^3 - 2x^2 + 1}$

Circle one:

The graph of  $f$  has a horizontal asymptote.

The graph of  $f$  has a slant asymptote.

The graph of  $f$  does not have a horizontal or slant asymptote.

6.  $f(x) = \frac{2x^4 + x^2 + 1}{3x^4 - 2x^2 + 5x}$

Circle one:

The graph of  $f$  has a horizontal asymptote.

The graph of  $f$  has a slant asymptote.

The graph of  $f$  does not have a horizontal or slant asymptote.

7.  $f(x) = \frac{x^3 + 5x^2 + x + 2}{3x^4 - 2x^3 + 2x^2 - 3}$

Circle one:

The graph of  $f$  has a horizontal asymptote.

The graph of  $f$  has a slant asymptote.

The graph of  $f$  does not have a horizontal or slant asymptote.

Write the equation for the slant asymptote for the following functions.

8.  $f(x) = \frac{x^3 - 2x^2 - 4x + 1}{x^2 - 2x + 1}$

$$\begin{array}{r} x^3 - 2x^2 + 1 \overline{) x^3 - 2x^2 - 4x + 1} \\ \underline{-(x^3 - 2x^2 + 1x)} \phantom{+ 1} \\ -5x + 1 \phantom{+ 1} \end{array}$$

$y = x$

9.  $f(x) = \frac{x^2 - 9x + 4}{x + 6}$

$$\begin{array}{r|rrr} -6 & 1 & -9 & 4 \\ & & -6 & 90 \\ \hline & 1 & -15 & 94 \end{array}$$

$y = x - 15$

10.  $f(x) = \frac{4x^2 + 12x - 6}{2x + 1}$

must divide numerator by 2

$$\begin{array}{r} 4 \quad 12 \quad -6 \\ \frac{1}{2} \overline{) 2 \quad 6 \quad -3} \\ \underline{2 \quad 5} \phantom{-3} \\ -5.5 \end{array}$$

Note: If you needed the remainder, it would be  $-5.5(2) = -11$

$y = 2x + 5$

11.  $f(x) = \frac{9x^4 - 5x^2 + 3x - 6}{3x^3 - 4x^2}$

$$\begin{array}{r} 3x + 4 \overline{) 9x^4 + 0x^3 - 5x^2 + 3x - 6} \\ \underline{-(9x^4 - 12x^3)} \phantom{- 6} \\ 12x^3 - 5x^2 + 3x - 6 \\ \underline{-(12x^3 - 16x^2)} \phantom{+ 3x - 6} \\ 11x^2 + 3x - 6 \end{array}$$

$y = 3x + 4$

Use the rational function to answer the following.

12.  $f(x) = \frac{3x(x+2)(x-2)}{(x-4)(x+2)}$

d. Vertical Asymptote(s):  $x = 4$

g. y-intercept:  $\frac{0}{-8} = 0$

a. Domain:  $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$

e. Horizontal Asymptote: none

h. x-intercept(s):  $x = 0$  and  $2$

b. Zero(s):  $x = 0$  and  $2$

f. Slant Asymptote:  $y = 3x + 6$

i. End Behavior:  $\lim_{x \rightarrow -\infty} f(x) = -\infty$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$

c. Hole(s):  $x = -2$

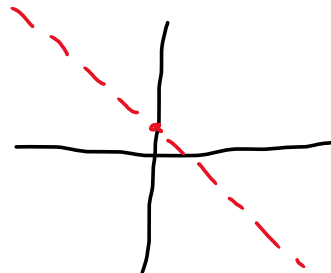
$$\begin{array}{r} 3x + 6 \overline{) 3x^3 + 0x^2 - 12x} \\ \underline{-(3x^3 - 6x^2 - 24x)} \phantom{+ 6} \\ 6x^2 + 12x \\ \underline{-(6x^2 - 12x - 8)} \phantom{+ 6} \\ 24x + 8 \end{array}$$

Multiple Choice

13. The function  $f$  is a rational function. The quotient and remainder form of  $f$  is given by  $f(x) = -2x + 1 + \frac{3x+4}{x^2-4x-12}$ . Which describes the end behavior of  $f$ ?

slant asymptote

- (A)  $\lim_{x \rightarrow -\infty} f(x) = -\infty$  and  $\lim_{x \rightarrow \infty} f(x) = -\infty$
- (B)  $\lim_{x \rightarrow -\infty} f(x) = \infty$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$
- (C)  $\lim_{x \rightarrow -\infty} f(x) = -\infty$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$
- (D)  $\lim_{x \rightarrow -\infty} f(x) = \infty$  and  $\lim_{x \rightarrow \infty} f(x) = -\infty$



14. Which of the following is equivalent to  $\frac{x^2+5x+2}{x+5}$  ?

- (A)  $x + 1$
- (B)  $x + 2$
- (C)  $x + \frac{2}{x+5}$
- (D)  $x + 1 - \frac{4}{x+5}$

$$\begin{array}{r}
 -5 \overline{) 1 \ 5 \ 2} \\
 \underline{1 \ 5 \ 0} \\
 1 \ 0 \ 2 \\
 1 \ 0 \ 0 \\
 \hline
 2 \\
 x + 0 + \frac{2}{x+5}
 \end{array}$$

15. The function  $f$  is given by  $f(x) = \frac{6x^2+ax+2}{x+3}$  and has a slant asymptote of  $y = 6x + 3$ . What is the value of  $a$ ?

- (A) -4
- (B) 12
- (C) 15
- (D) 21

$$\begin{array}{r}
 -3 \overline{) 6 \ a \ 2} \\
 \underline{6 \ 18} \\
 6 \ 3
 \end{array}$$

$a = 21$