AP Precalc

Write your questions and thoughts here!

# 1.5A Polynomial Functions and Complex Zeros

**1.5A Notes**

Today, we will examine **complex zeros** of a polynomial function. *Complex* refers to both **real** numbers and **non-real** numbers.

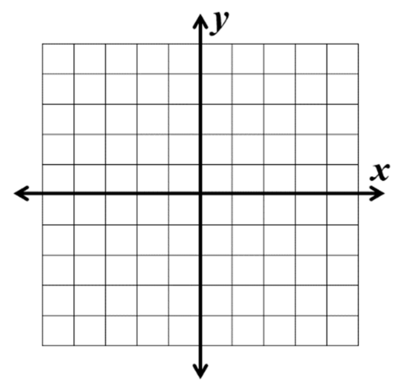
## Zeros, $x$-intercepts, and Linear Factors

If we have $p(a) = 0$, then

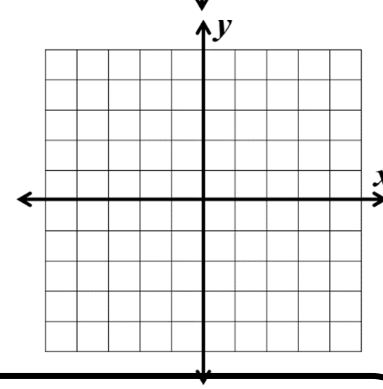
- …$a$ is called a ________________ of $p$.
- …we have an _________________ at the point __________.
- …if $a$ is a real number, then __________ is a _____________________ of $p$.

We are not reviewing all the factoring techniques from Algebra 2, but you will need to remember some basic factoring of quadratics. Here's an example.

1. Find all the $x$-intercepts of the function $g(x) = x^3 - 2x^2 - 8x$.



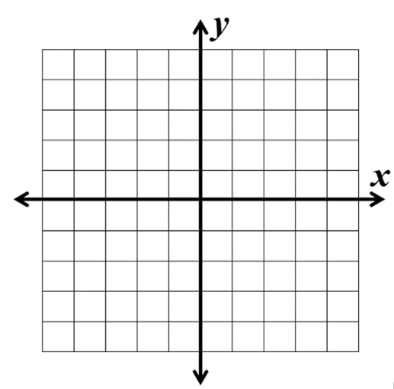

2. Given the polynomial function $h(x) = x^2 - 2x - 3$, what are all intervals on which $h(x) \geq 0$?




## Multiplicity

If a linear factor $(x - a)$ is repeated $n$ times, the corresponding zero of the polynomial function has a multiplicity $n$.

$$p(x) = (x + 1)^2(x - 3)^3(x + 4)$$




If the real zero, $a$, has even multiplicity, then the graph will "bounce" off the $x$-axis at $x = a$.

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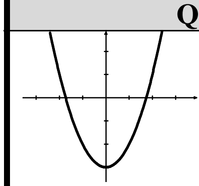
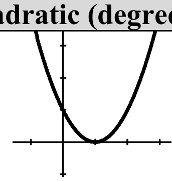
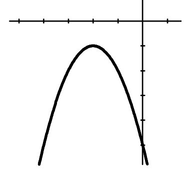
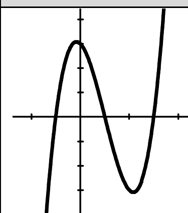
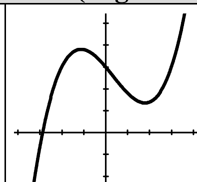
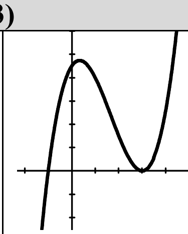
Another way of thinking about this is to look at the output values for input values near $x = a$. The output values will all have the same sign (positive or negative).

3. Given the polynomial function $p(x) = (x-2)(x-5)^4(x+7)$, what are all intervals on which $p(x) \leq 0$?

## Complex Zeros

A polynomial of degree $n$ has exactly $n$ complex zeros when counting multiplicities. "Complex" refers to both real and non-real zeros.

If there are any non-real zeros, they always come in conjugate pairs (see explanation below). This means there are either no non-real zeros, or an even amount of non-real zeros.

| **Quadratic (degree = 2)** | | | **Cubic (degree = 3)** | | |
|---|---|---|---|---|---|
|  |  |  |  |  |  |
| # of real 0's | # of real 0's | # of real 0's | # of real 0's | # of real 0's | # of real 0's |
| # of non-real 0's | # of non-real 0's | # of non-real 0's | # of non-real 0's | # of non-real 0's | # of non-real 0's |

4. The degree of a polynomial is 8 with real zeros at $x = -10, x = 5$, and $x = 16$. $x = 5$ has a multiplicity of 2. How many non-real zeros does the polynomial have?

## Non-real Zeros

If $a + bi$ is a non-real zero of a polynomial $p$, then its conjugate __________ is also a zero of $p$.

| **Given one non-real zero of a polynomial, find another zero.** | |
|---|---|
| 5. $-3 + 6i$ | 6. $4 - 2i$ |

Write your questions and thoughts here!

## Successive differences to find the degree of a polynomial.

If you have input and output values of a polynomial function, it is possible to find the degree of the function. This technique only works if the input values are over equal intervals. The degree of the polynomial function is equal to the least value $n$ for which the successive $n$th differences are constant.

7. Find the degree of the polynomial from the given input and output values.

| **Input** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **Output** | 0 | 1 | 4 | 9 | 16 | 25 | 36 | 49 |

8. Find the degree of the polynomial from the given input and output values.

| **Input** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **Output** | 50 | 49 | 38 | 5 | $-62$ | $-175$ | $-346$ | $-587$ |

## 1.5A Polynomial Functions and Complex Zeros

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**1.5A Practice**

**For each polynomial function, find the intervals for each condition.**

1. $f(x) = x^2 - 5x + 4$. When is $f(x) \leq 0$?

2. $g(x) = x^2 + 17x + 70$. When is $g(x) \geq 0$?

3. $p(x) = (x-7)(x-1)^2$. When is $p(x) \leq 0$?

4. $h(x) = x^3 + 9x^2 + 18x$. When is $h(x) \geq 0$?

5. $a(x) = x(x-8)^3(x+3)^4$. When is $a(x) \geq 0$?

6. $f(x) = -x(x+4)^2(x+1)(x-6)^6$. When is $f(x) \leq 0$?

**For each polynomial, the degree is listed along with all of its real zeros. Find the number of NON-REAL zeros the polynomial has.**

7. The degree is 5 with real zeros at $x = -5, x = 1$, and $x = 4$.

8. The degree is 6 with real zeros at $x = -12$ and $x = 7$.

9. The degree is 8 with real zeros at $x = 0, x = 2$, and $x = 3$. $x = 2$ has a multiplicity of 4.

10. The degree is 16 with real zeros at $x = -2$ and $x = 10$. $x = -2$ has a multiplicity of 5.

11. The degree is 12 with real zeros at $x = 14, x = -6$, and $x = -10$. $x = 14$ has a multiplicity of 6.

12. The degree is 50 with real zeros at $x = 7$ and $x = 8$. $x = 8$ has a multiplicity of 19.

**Given one non-real zero of a polynomial, find another zero.**

| | | | |
|---|---|---|---|
| 13. $7 + 2i$ | 14. $-5 + i$ | 15. $1 - 5i$ | 16. $-3 - 4i$ |

**Find the degree of the polynomial from the given input and output values.**

17.

| **Input** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **Output** | $-2$ | $-32$ | $-162$ | $-512$ | $-1250$ | $-2592$ | $-4802$ | $-8192$ |

18.

| **Input** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **Output** | 6 | 24 | 54 | 96 | 150 | 216 | 294 | 384 |

19.

| **Input** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **Output** | 4 | 1 | 6 | 127 | 688 | 2,349 | 6,226 | 14,011 |

20.

| Input | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Output | $-10$ | $-6$ | $8$ | $44$ | $114$ | $230$ | $404$ | $648$ |

## 1.5A Polynomial Functions and Complex Zeros

**1.5A Test Prep**

21. A polynomial function has 3 real zeros and 4 non-real zeros. One of the real zeros has a multiplicity of 6. What is the degree of the polynomial?

(A) 7 (B) 9 (C) 12 (D) 13

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22. **No calculator allowed!** The polynomial function $g$ is given by $g(x) = (x-6)(x^2+2x+2)$. Which of the following describes the zeros of $g$?

(A) $g$ has exactly two distinct real zeros.

(B) $g$ has exactly three distinct real zeros.

(C) $g$ has exactly one distinct real zero and no non-real zeros.

(D) $g$ has exactly one distinct real zero and two non-real zeros.