1.5A Polynomial Functions and Complex Zeros

AP Precalculus

1.5A Practice

For each polynomial function, find the intervals for each condition.

1. $f(x) = x^2 - 5x + 4$. When is $f(x) \leq 0$?

$0 = (x-4)(x-1)$

$x-4=0$ $x-1=0$

$x=4$ $x=1$

x	$(-\infty, 1)$	1	$(1,4)$	4	$(4,\infty)$
$f(x)$	pos.	0	neg.	0	pos.

$f(x) \leq 0$ on the interval $[1, 4]$.

2. $g(x) = x^2 + 17x + 70$. When is $g(x) \geq 0$?

$0 = (x+7)(x+10)$

$x=-7$ $x=-10$

x	$(-\infty, -10)$	-10	$(-10,-7)$	-7	$(-7,\infty)$
$f(x)$	pos.	0	neg	0	pos

$g(x) \geq 0$ on the interval $(-\infty, -10] \cup [-7, \infty)$.

3. $p(x) = (x-7)(x-1)^2$. When is $p(x) \leq 0$?

$x = 7 \quad x = 1$

even mult.

x	$(-\infty, 1)$	1	$(1, 7)$	7	$(7, \infty)$
$f(x)$	neg	0	neg	0	pos

$p(x) \leq 0$ on the interval $(-\infty, 7]$.

4. $h(x) = x^3 + 9x^2 + 18x$. When is $h(x) \geq 0$?

$0 = x(x^2 + 9x + 18)$

$0 = x(x+3)(x+6)$

$x = 0 \quad x = -3 \quad x = -6$

x	$(-\infty, -6)$	-6	$(-6, -3)$	-3	$(-3, 0)$	0	$(0, \infty)$
$f(x)$	neg	0	pos	0	neg	0	pos

$h(x) \geq 0$ on the interval $[-6, -3] \cup [-0, \infty)$.

5. $a(x) = x(x-8)^3(x+3)^4$. When is $a(x) \geq 0$?

$x = 0, \quad x = 8, \quad x = -3$

even mult.

x	$(-\infty, -3)$	-3	$(-3, 0)$	0	$(0, 8)$	8	$(8, \infty)$
$f(x)$	pos	0	pos	0	neg	0	pos

$a(x) \geq 0$ on the interval $(-\infty, 0] \cup [8, \infty)$.

6. $f(x) = -x(x+4)^2(x+1)(x-6)^6$. When is $f(x) \leq 0$?

$x = 0 \quad x = -4 \quad x = -1 \quad x = 6$

even mult   even mult.

x	$(-\infty, -4)$	-4	$(-4, -1)$	-1	$(-1, 0)$	0	$(0, 6)$	6	$(6, \infty)$
$f(x)$	neg	0	neg	0	pos	0	neg	0	neg

$f(x) \leq 0$ on the interval $(-\infty, -1] \cup [0, \infty)$.

For each polynomial, the degree is listed along with all of its real zeros. Find the number of NON-REAL zeros the polynomial has.

7. The degree is 5 with real zeros at $x = -5, x = 1$, and $x = 4$.

5 - 3 real =

2 non-real zeros

8. The degree is 6 with real zeros at $x = -12$ and $x = 7$.

6 - 2 real =

4 non-real zeros

9. The degree is 8 with real zeros at $x = 0, x = 2$, and $x = 3$. $x = 2$ has a multiplicity of 4.

$8 - 2 - 4 =$

2 non-real zeros

10. The degree is 16 with real zeros at $x = -2$ and $x = 10$. $x = -2$ has a multiplicity of 5.

$16 - 1 - 5 =$

10 non-real zeros

11. The degree is 12 with real zeros at $x = 14, x = -6$, and $x = -10$. $x = 14$ has a multiplicity of 6.

$12 - 2 - 6 =$

4 non-real zeros

12. The degree is 50 with real zeros at $x = 7$ and $x = 8$. $x = 8$ has a multiplicity of 19.

$50 - 1 - 19$

30 non-real

Given one non-real zero of a polynomial, find another zero.

13. $7 + 2i$	14. $-5 + i$	15. $1 - 5i$	16. $-3 - 4i$
$\boxed{7 - 2i}$	$\boxed{-5 - i}$	$\boxed{1 + 5i}$	$\boxed{-3 + 4i}$

Find the degree of the polynomial from the given input and output values.

17.

Input	0	1	2	3	4	5	6	7
Output	−2	−32	−162	−512	−1250	−2592	−4802	−8192

1st difference → −30 −130 −350 −738 −1342 −2210 −3390

2nd difference → −100 −220 −388 −604 −868 −1180

3rd difference → −120 −168 −216 −264 −312

4th difference → −48 −48 −48 −48

$\boxed{\text{Degree} = 4}$

18.

Input	0	1	2	3	4	5	6	7
Output	6	24	54	96	150	216	294	384

1st difference → 18 30 42 54 66 78 90

2nd difference → 12 12 12 12 12 12

$\boxed{\text{Degree} = 2}$

19.

Input	0	1	2	3	4	5	6	7
Output	4	1	6	127	688	2,349	6,226	14,011

1st difference → −3 5 121 561 1661 3877 7785

2nd difference → 8 116 440 1100 2216 3908

3rd difference → 108 324 660 1116 1692

4th difference → 216 336 456 576

5th difference → 120 120 120

$\boxed{\text{Degree} = 5}$

20.

Input	0	1	2	3	4	5	6	7
Output	−10	−6	8	44	114	230	404	648

1st difference → 4 14 36 70 116 174 244

2nd difference → 10 22 34 46 58 70

3rd difference → 12 12 12 12 12

$\boxed{\text{Degree} = 3}$

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1.5A Test Prep

21. A polynomial function has 3 real zeros and 4 non-real zeros. One of the real zeros has a multiplicity of 6. What is the degree of the polynomial?

C

2 real zeros = 2
1 real w/ multiplicity 6 = 6
4 non-real = 4

2+6+4 = 12

(A) 7 (B) 9 (C) 12 (D) 13

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22. **No calculator allowed!** The polynomial function g is given by $g(x) = (x-6)(x^2+2x+2)$. Which of the following describes the zeros of g?

x = 6 is a zero.

Does not factor. Check the zeros using the quadratic formula.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$x = \frac{-2 \pm \sqrt{-4}}{2}$$

non-real!

Two non-real zeros

(A) g has exactly two distinct real zeros.

(B) g has exactly three distinct real zeros.

(C) g has exactly one distinct real zero and no non-real zeros.

(D) g has exactly one distinct real zero and two non-real zeros.