AP Precalc

Write your questions and thoughts here!

1.7B Rational Functions & End Behavior

1.7B Notes

End Behavior

Example 1:

$$f(x) = \frac{2x + 3}{x^2 - 4x - 32}$$

End Behavior:

$$\lim_{x \to -\infty} f(x) =$$

$$\lim_{x \to \infty} f(x) =$$

For inputs of large magnitude,
the polynomial in the denominator _____ the polynomial in the numerator.

Example 2:

$$g(x) = \frac{x^3 - 3x + 5}{5x^2 + x + 12}$$

End Behavior:

Example 3:

$$h(t) = \frac{2t + 3}{t}$$

End Behavior:

Finding Horizontal Asymptotes!

Horizontal asymptotes =

- Degree of denominator is bigger:
- Degree of numerator is bigger:
- Degrees are the same:

Degree on bottom is bigger!	Degree on top is bigger!	Degrees are the same!
$f(x) = \frac{4x^2 + x + 8}{x^3 + 3x^2 - 1}$	$f(x) = \frac{x^2 + 2x - 15}{4x - 9}$	$f(x) = \frac{x^2 + 5x - 36}{3x^2 + 21x}$
Horizontal Asymptote:	Horizontal Asymptote:	Horizontal Asymptote:

Write your questions and thoughts here!

Horizontal Asymptote

Let f be the rational function

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + \cdots + b_2 x^2 + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors.

a. When $n < m$, the graph of f has the line $y = 0$ as a horizontal asymptote.

b. When $n > m$, the graph of f has no horizontal asymptote.

c. When $n = m$, the graph of f has the line $y = \frac{a_n}{b_n}$ as a horizontal asymptote.

Limits

Evaluate the following limits.

Example 4:

$$\lim_{x\to\infty} \frac{(x+4)(2x-5)}{x^2-9} =$$

Example 5:

$$\lim_{x\to-\infty} \frac{3x^3+2}{x^2-9} =$$

Quick Practice

1. Find the end behavior.

$$d(t) = \frac{t^2 + 2t - 15}{4t - 9}$$

2. Find the horizontal asymptote if one exists.

$$g(x) = \frac{x^4 + 3x^2 - 5}{4x^5 - x^3 + 2x}$$

3. Evaluate the limit.

$$\lim_{n\to-\infty} \frac{3n^5+2}{n^3-9} =$$

1.7B Rational Functions and End Behavior

AP Precalculus

1.7B Practice

Determine the end behavior of the following. Determine the horizontal asymptote if one exists.

1. $f(x) = \frac{x^2-3x+1}{2x+3}$

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

2. $A(r) = \frac{3r^2-4}{5r^3+3r+2}$

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

3. $h(x) = \frac{x^2-9}{4x^2+2x-15}$

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

4. $g(n) = \frac{3n^2-4}{(n+3)(n-7)}$

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

5. $f(x) = \frac{-2x^3+5}{4x^5-8x^3+2x}$

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

6. $g(x) = \frac{-3x^4+x^2+2}{2x^4+7x^2-5}$

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

Find the horizontal asymptote of the following rational function if one exists.

7. $f(x) = \frac{x-1}{x}$

8. $d(t) = \frac{4-t^2}{2t^2+t}$

9. $h(x) = \frac{x^2-1}{4x(x^2+1)}$

Evaluate the following limits.

10. $\lim_{x\to\infty} \frac{3x^2+2}{x^2-9} =$	11. $\lim_{x\to-\infty} \frac{1}{x-4} =$	12. $\lim_{n\to\infty} \frac{2n^5-3n^3+2n}{5n^5+3n-2} =$
13. $\lim_{x\to\infty} \frac{(x+3)(2x-1)}{4x^2-9} =$	14. $\lim_{t\to-\infty} \frac{2t^3-3t}{5t^2-4t} =$	15. $\lim_{n\to-\infty} \frac{n^3}{2n^3-2n} =$

Use the rational function to answer the following.

16. $f(x) = \frac{x^2-2x-24}{2x^3+10x^2}$

a. Domain:

b. y-intercept:

c. $\lim_{x\to-\infty} f(x) =$

d. $\lim_{x\to\infty} f(x) =$

e. As x increases without bound the $f(x)$…

f. As x decreases without bound the $f(x)$…

g. **Multiple Choice** Which of the following is true for input values of large magnitude?

(A) The polynomial in the numerator dominates the polynomial in the denominator indicating no horizontal asymptote.

(B) The polynomial in the numerator dominates the polynomial in the denominator indicating a horizontal asymptote of $y = 0$.

(C) The polynomial of the denominator dominates the polynomial in the numerator indicating no horizontal asymptote.

(D) The polynomial of the denominator dominates the polynomial in the numerator indicating a horizontal asymptote of $y = 0$.

(E) Neither polynomial of the rational function dominates the other indicating a horizontal asymptote of $y = \frac{1}{2}$

17. $g(x) = \frac{4(x+5)(x-2)}{x^2-9}$

a. Domain:

b. y-intercept:

c. $\lim_{x\to-\infty} g(x) =$

d. $\lim_{x\to\infty} g(x) =$

e. As x increases without bound the $g(x)$…

f. As x decreases without bound the $g(x)$…

g. **Multiple Choice** Which of the following is true for input values of large magnitude?

(A) The polynomial in the numerator dominates the polynomial in the denominator indicating no horizontal asymptote.

(B) The polynomial in the numerator dominates the polynomial in the denominator indicating a horizontal asymptote of $y = 0$.

(C) The polynomial of the denominator dominates the polynomial in the numerator indicating no horizontal asymptote.

(D) The polynomial of the denominator dominates the polynomial in the numerator indicating a horizontal asymptote of $y = 0$.

(E) Neither polynomial of the rational function dominates the other indicating a horizontal asymptote of $y = 4$.

Multiple Choice

18. The function f is given by $f(x) = \frac{(x-8)(x+3)}{x^2+5x-24}$. Which of the following describes the function f ?

(A) Domain is $(-\infty,-3) \cup (-3,8) \cup (8,\infty)$ with a horizontal asymptote of $y = 1$.

(B) Domain is $(-\infty,-8) \cup (-8,3) \cup (3,\infty)$ with a horizontal asymptote of $y = 1$.

(C) Domain is $(-\infty,-3) \cup (-3,8) \cup (8,\infty)$ with a horizontal asymptote of $y = 0$.

(D) Domain is $(-\infty,-8) \cup (-8,3) \cup (3,\infty)$with a horizontal asymptote of $y = 0$.

(E) The function does not have a horizontal asymptote.

19. The function f is given by $f(x) = \frac{ax^3-2x^2+5}{2x^3-8}$ and has line $y = 3$ as a horizontal asymptote. Which of the following must be true?

(A) $f(a) = 6$

(B) $a = 6$

(C) $\lim_{x\to\infty} f(x) = a$

(D) $\lim_{x\to\infty} f(x) = 6$

(E) None of the above are true.

20. Which of the following is equivalent to $\lim_{x\to-\infty} \frac{3x^2+2x+6}{5x^4-9x^2+2}$?

(A) $\frac{3}{5}$

(B) $-\frac{3}{5}$

(C) 3

(D) 0

(E) Does not exist.

21. The function f is given by $f(x) = \frac{-x^3+3x^2+x-5}{5x^2+7x-4}$. Which describes the end behavior of f?

(A) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$

(B) $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$

(C) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

(D) $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$

(E) $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$