1.7B Rational Functions and End Behavior

AP Precalculus

1.7B Practice

Determine the end behavior of the following. Determine the horizontal asymptote if one exists.

1. $f(x) = \frac{x^2-3x+1}{2x+3}$

End Behavior:

Degree of numerator dominates!

$\frac{(-\infty)^2}{-\infty} = \frac{+}{-}$ $\frac{(\infty)^2}{\infty} = \frac{+}{+}$

$\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

nope

2. $A(r) = \frac{3r^2-4}{5r^3+3r+2}$

End Behavior:

Degree of denominator dominates!

$\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y = 0$

3. $h(x) = \frac{1x^2-9}{4x^2+2x-15}$

End Behavior:

Degrees are equal in numerator and denominator!

$\lim_{x\to-\infty} f(x) = \frac{1}{4}$ and $\lim_{x\to\infty} f(x) = \frac{1}{4}$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y = \frac{1}{4}$

4. $g(n) = \frac{3n^2-4}{(n+3)(n-7)}$

$n^2-4n-21$

End Behavior:

Degrees are equal in numerator and denominator!

$\lim_{x\to-\infty} f(x) = 3$ and $\lim_{x\to\infty} f(x) = 3$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y = 3$

5. $f(x) = \frac{-2x^3+5}{4x^5-8x^3+2x}$

End Behavior:

Degree of denominator dominates!

$\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y = 0$

6. $g(x) = \frac{-3x^4+x^2+2}{2x^4+7x^2-5}$

End Behavior:

Degrees are equal in numerator and denominator!

$\lim_{x\to-\infty} f(x) = \frac{-3}{2}$ and $\lim_{x\to\infty} f(x) = \frac{-3}{2}$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y = -\frac{3}{2}$

Find the horizontal asymptote of the following rational function if one exists.

7. $f(x) = \frac{x-1}{x}$

Degrees are equal in numerator and denominator!

$y = 1$

8. $d(t) = \frac{4-t^2}{2t^2+t}$

Degrees are equal in numerator and denominator!

$y = -\frac{1}{2}$

9. $h(x) = \frac{x^2-1}{4x(x^2+1)}$

Degree of denominator dominates!

$y = 0$

Evaluate the following limits.		
10. $\lim_{x\to\infty}\frac{3x^2+2}{x^2-9} = \frac{3}{1} = 3$	11. $\lim_{x\to-\infty}\frac{1}{x-4} = 0$	12. $\lim_{n\to\infty}\frac{2n^5-3n^3+2n}{5n^5+3n-2} = \frac{2}{5}$
13. $\lim_{x\to\infty}\frac{(x+3)(2x-1)}{4x^2-9} = \frac{2}{4} = \frac{1}{2}$ ($2x^2+5x-3$)	14. $\lim_{t\to-\infty}\frac{2t^3-3t}{5t^2-4t} = \frac{(-\infty)^3}{(-\infty)^2} = \frac{-}{+} = -\infty$	15. $\lim_{n\to-\infty}\frac{n^3}{2n^3-2n} = \frac{1}{2}$

Use the rational function to answer the following.

16. $f(x) = \frac{x^2-2x-24}{2x^3+10x^2}$ $\frac{(x-6)(x+4)}{2x^2(x+5)}$

a. Domain:

$(-\infty,-5)\cup(-5,0)\cup(0,\infty)$

b. y-intercept:

$f(0) = \frac{-24}{0} = undefined$

c. $\lim_{x\to-\infty} f(x) = 0$

d. $\lim_{x\to\infty} f(x) = 0$

e. As x increases without bound the $f(x)$…

approaches zero

f. As x decreases without bound the $f(x)$…

approaches zero

g. **Multiple Choice** Which of the following is true for input values of large magnitude?

(A) The polynomial in the numerator dominates the polynomial in the denominator indicating no horizontal asymptote.

(B) The polynomial in the numerator dominates the polynomial in the denominator indicating a horizontal asymptote of $y = 0$.

(C) The polynomial of the denominator dominates the polynomial in the numerator indicating no horizontal asymptote.

(D) The polynomial of the denominator dominates the polynomial in the numerator indicating a horizontal asymptote of $y = 0$.

(E) Neither polynomial of the rational function dominates the other indicating a horizontal asymptote of $y = \frac{1}{2}$

17. $g(x) = \frac{4(x+5)(x-2)}{x^2-9}$ $4(x^2+3x-10) = 4x^2+12-40$ $(x+3)(x-3)$

a. Domain:

$(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

b. y-intercept:

$f(0) = \frac{-40}{-9} = \frac{40}{9}$

c. $\lim_{x\to-\infty} g(x) = \frac{4}{1} = 4$

d. $\lim_{x\to\infty} g(x) = \frac{4}{1} = 4$

e. As x increases without bound the $g(x)$…

approaches four

f. As x decreases without bound the $g(x)$…

approaches four

g. **Multiple Choice** Which of the following is true for input values of large magnitude?

(A) The polynomial in the numerator dominates the polynomial in the denominator indicating no horizontal asymptote.

(B) The polynomial in the numerator dominates the polynomial in the denominator indicating a horizontal asymptote of $y = 0$.

(C) The polynomial of the denominator dominates the polynomial in the numerator indicating no horizontal asymptote.

(D) The polynomial of the denominator dominates the polynomial in the numerator indicating a horizontal asymptote of $y = 0$.

(E) Neither polynomial of the rational function dominates the other indicating a horizontal asymptote of $y = 4$.

Multiple Choice

18. The function f is given by $f(x) = \frac{1(x-8)(x+3)}{x^2+5x-24}$. Which of the following describes the function f ?

$1(x+8)(x-3) \neq 0$

(A) Domain is $(-\infty,-3) \cup (-3,8) \cup (8,\infty)$ with a horizontal asymptote of $y = 1$.

(B) Domain is $(-\infty,-8) \cup (-8,3) \cup (3,\infty)$ with a horizontal asymptote of $y = 1$.

(C) Domain is $(-\infty,-3) \cup (-3,8) \cup (8,\infty)$ with a horizontal asymptote of $y = 0$.

(D) Domain is $(-\infty,-8) \cup (-8,3) \cup (3,\infty)$with a horizontal asymptote of $y = 0$.

(E) The function does not have a horizontal asymptote.

19. The function f is given by $f(x) = \frac{ax^3-2x^2+5}{2x^3-8}$ and has line $y = 3$ as a horizontal asymptote. Which of the following must be true?

(A) $f(a) = 6$

(B) $a = 6$

$$\frac{6x^3-2x^2+5}{2x^3-8} = \frac{6}{2} = 3$$

(C) $\lim_{x\to\infty} f(x) = a$

(D) $\lim_{x\to\infty} f(x) = 6$

(E) None of the above are true.

20. Which of the following is equivalent to $\lim_{x\to-\infty} \frac{3x^2+2x+6}{5x^4-9x^2+2}$?

(A) $\frac{3}{5}$

(B) $-\frac{3}{5}$

(C) 3

(D) 0 The degree of the denominator dominates!

(E) Does not exist.

21. The function f is given by $f(x) = \frac{-x^3+3x^2+x-5}{5x^2+7x-4}$. Which describes the end behavior of f?

(A) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$

left: $\frac{-(-\infty)^3}{(-\infty)^2} = \frac{-(-)}{+} = \frac{+}{+} = \infty$

(B) $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$

(C) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

right: $\frac{-(\infty)^3}{(\infty)^2} = \frac{-(+)}{+} = \frac{-}{+} = -\infty$

(D) $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$

(E) $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$