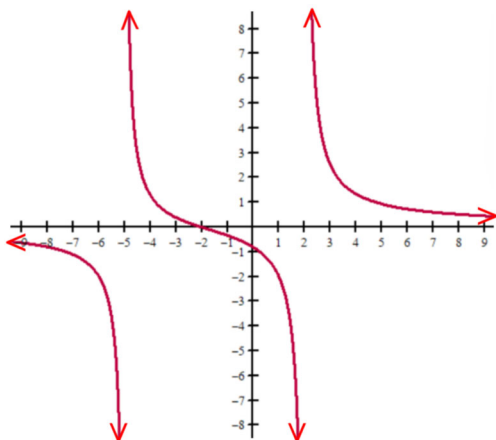


Write your questions
and thoughts here!

Vertical Asymptotes

Example 1:



Vertical Asymptote(s):

Limit Notation:

$$\lim_{x \rightarrow -5^-} f(x) =$$

As x approaches -5 from
the left the $f(x)$...

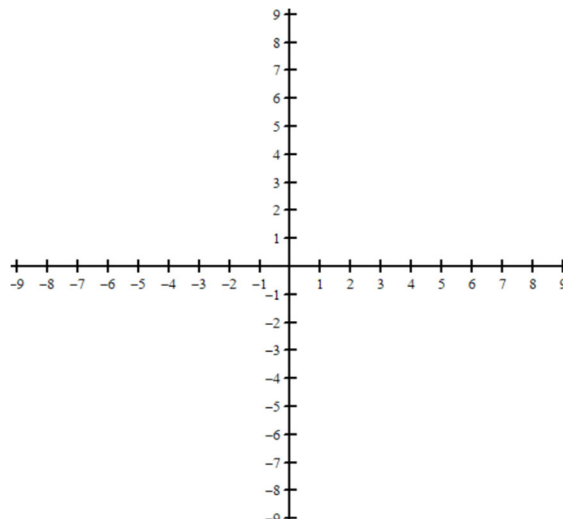
$$\lim_{x \rightarrow -5^+} f(x) =$$

As x approaches -5 from
the right the $f(x)$...

Example 2:

$$g(x) = \frac{x^2 - 9}{x^2 - 4}$$

Limit Notation of Vertical Asymptote(s):

Vertical AsymptotesLet f be the rational function

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + b_{m-2} x^{m-2} + \dots + b_2 x^2 + b_1 x + b_0}$$

where $N(x)$ and $D(x)$ have no common factors.The graph of f has vertical asymptotes at the zeros of $D(x)$.

Write your questions
and thoughts here!



Numerically

Example 3:

$$g(x) = \frac{x^2 + 3x}{x - 2}$$

As x approaches 2 from the left
the $g(x)$...

x	$g(x)$
1.9	
1.99	
1.999	
1.9999	

As x approaches 2 from the right
the $g(x)$...

x	$g(x)$
2.1	
2.01	
2.001	
2.0001	

Multiplicity

Example 4:

$$h(x) = \frac{x^2 + 4x - 12}{(x - 2)^2}$$

Domain:

Vertical Asymptote(s):

Hole(s):

Limit Notation of Vertical Asymptote(s):

Zero(s):

Horizontal Asymptote:

End Behavior:

1.9 Rational Functions and Vertical Asymptotes

AP Precalculus

1.9 Practice

Find the domain and vertical asymptote(s) of the following rational function if one exists.

1. $f(x) = \frac{x(x+2)}{x^2-4}$

Domain:

Vertical Asymptote(s):

2. $d(t) = \frac{t^2+4t-12}{(t-2)^2}$

Domain:

Vertical Asymptote(s):

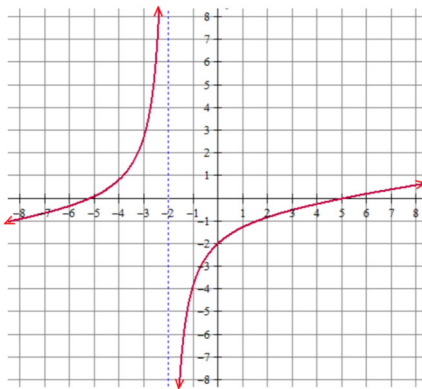
3. $h(x) = \frac{x^3-3x^2}{x^2+8x+15}$

Domain:

Vertical Asymptote(s):

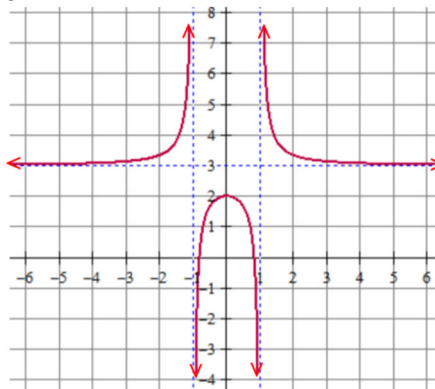
State the vertical asymptote(s) of the following rational functions. Use limit notation.

4.



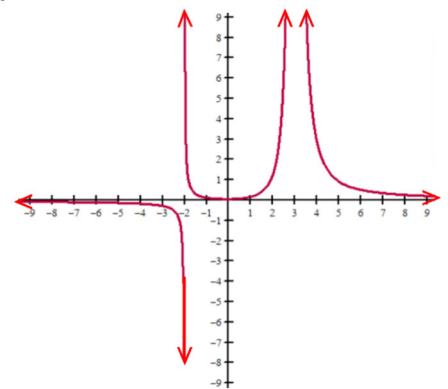
Limit Notation Vertical Asymptote(s):

5.



Limit Notation Vertical Asymptote(s):

6.



Limit Notation Vertical Asymptote(s):

CALCULATOR ACTIVE Complete the table to answer the following.

7. $f(x) = \frac{x^2-1}{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$							

Vertical Asymptote:

Limit Notation of Vertical Asymptote:

CALCULATOR ACTIVE Complete the table to answer the following.

8. $f(x) = \frac{x^2 - 2x}{x + 2}$

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$							

Vertical Asymptote:

Limit Notation of Vertical Asymptote:

Use the table of the rational function h to find the following.

9.

t	$d(t)$
-0.1	5,589
-0.01	37,231
-0.001	96,543
-0.0001	148,234
0	undefined
0.0001	128,341
0.001	89,437
0.01	18,235
0.1	1,455

a. Find $d(0) =$

b. Find the y -intercept.

c. Find $\lim_{t \rightarrow 0^-} d(t) =$

d. Find $\lim_{t \rightarrow 0^+} d(t) =$

e. As t approaches zero from the left the $d(t) \dots$

f. As t approaches zero from the right the $d(t) \dots$

Make a sketch of the rational function with the following characteristics.

10. The graph of f has...

a. $f(-4) = 0$

b. $f(6) = 0$

c. $\lim_{x \rightarrow -3^-} f(x) = -\infty$

d. $\lim_{x \rightarrow -3^+} f(x) = \infty$

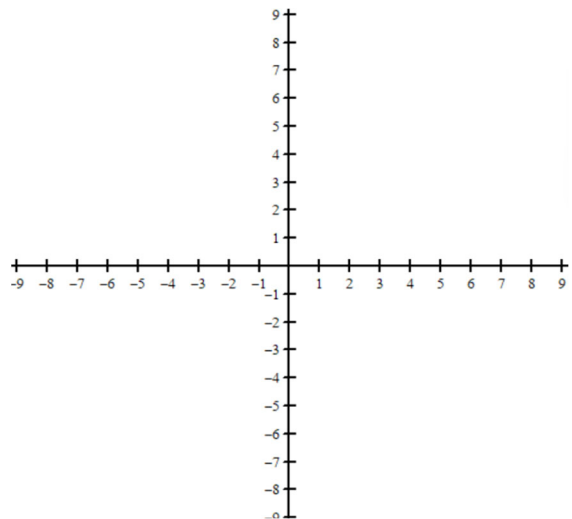
e. $\lim_{x \rightarrow 4^-} f(x) = \infty$

f. $\lim_{x \rightarrow 4^+} f(x) = -\infty$

g. $\lim_{x \rightarrow -\infty} f(x) = 2$

h. $\lim_{x \rightarrow \infty} f(x) = 2$

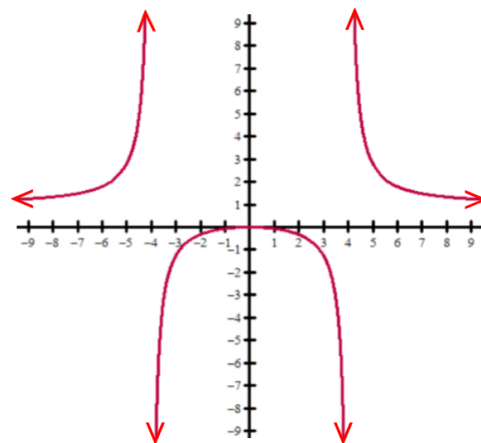
i. $f(0) = 5$



Multiple Choice

11. Given the graph of f . Which of the following describes the function f ?

- (A) $\lim_{x \rightarrow -4^-} f(x) = -\infty$ and $\lim_{x \rightarrow -4^+} f(x) = -\infty$
 (B) $\lim_{x \rightarrow -4^-} f(x) = \infty$ and $\lim_{x \rightarrow -4^+} f(x) = -\infty$
 (C) $\lim_{x \rightarrow -4^-} f(x) = -\infty$ and $\lim_{x \rightarrow -4^+} f(x) = \infty$
 (D) $\lim_{x \rightarrow -4^-} f(x) = \infty$ and $\lim_{x \rightarrow -4^+} f(x) = \infty$
 (E) $\lim_{x \rightarrow -4} f(x) = f(0)$



Free Response

12. The function f is a rational function graphed in the xy -plane. The polynomial in the numerator of f has exactly one real zero at $x = 3$. The polynomial of the denominator of f has exactly two real zeros at both $x = 3$ and $x = 6$. The multiplicities of the zeros at $x = 3$ in the numerator and in the denominator are equal.

- Find the domain for the graph of f .
- Describe any holes and/or vertical asymptotes for the graph of f .
- Explain how your answer from part b would change if the multiplicities of the zeros at $x = 3$ in the numerator and denominator were not equal?