2.2 Change in Linear and Exponential Functions

AP Precalculus

2.2 Practice

A function has the following coordinate points. Could the function represent a linear function, exponential function, or neither?

1. $(1,2), (2,8), (3,20)$

no → $+6$ $+12$

no → $\times 4$ $\times \frac{5}{2}$

Neither

2. $(10,30), (11,20), (12,10)$

Yes → -10 -10

Linear

3. $(13,20), (14,4), \left(15, \frac{4}{5}\right)$

no → -16 $-?$

Yes → $\times \frac{1}{5}$ $\times \frac{1}{5}$

Exponential

4. $(7,6), (8,12), (9,20)$

no → $+6$ $+8$

no → $\times 2$ $\times \frac{5}{3}$

Neither

The following functions are either linear or exponential. Which is it? Justify your answer.

5.

x changes: 4, 4

x	1	5	9
$f(x)$	2	6	18

$f(x)$ changes: $+4$, $\times 3$; $+12$, $\times 3$

Exponential because for each input change of 4, f changes **proportionally** by a ratio of 3.

6.

x changes: 2, 2

x	-4	-2	0
$f(x)$	5	1	-3

$f(x)$ changes: -4, -4

Linear because for each input change of 2, f has a **constant rate of change** of -4.

7.

x changes: 3, 3

x	7	10	13
$f(x)$	5	105	205

$f(x)$ changes: $+100$, $+100$

Linear because for each input change of 3, f has a **constant rate of change** of 100.

8.

x changes: 9, 9

x	11	20	29
$f(x)$	8	4	2

$f(x)$ changes: -4, $\times \frac{1}{2}$; -2, $\times \frac{1}{2}$

Exponential because for each input change of 9, f changes **proportionally** by a ratio of $\frac{1}{2}$.

Is each function linear or exponential. Identify the constant (slope or ratio) that causes the output values to change?

9. $y = -\frac{1}{4} \cdot 7^x$

Exponential, ratio of 7.

10. $y = 4x - 6$

Linear, slope of 4.

11. $y = 2 - 9x$

Linear, slope of -9.

12. $y = \frac{1}{3} \cdot \left(\frac{2}{5}\right)^x$

Exponential, ratio of $\frac{2}{5}$.

13. $y - 7 = -5(x + 4)$

Linear, slope of -5.

14. $y + 1 = \left(\frac{3}{4}\right)^{x+7}$

Exponential, ratio of $\frac{3}{4}$.

15. $y + 5 = \frac{1}{6} \cdot 2^{x-3}$

Exponential, ratio of 2.

16. $y - 6 = 3(x - 10)$

Linear, slope of 3.

It is known that y is a linear function and that it passes through the given points. Write an equation for this function.

17. $(1, 4)$ and $(3, 10)$

$m = \frac{10-4}{3-1} = \frac{6}{2} = 3$

$y = 3(x-1)+4$
or
$y = 3(x-3)+10$

18. $(4, 15)$ and $(10, 3)$

$m = \frac{3-15}{10-4} = \frac{-12}{6} = -2$

$y = -2(x-4)+15$
or
$y = -2(x-10)+3$

19. $(10, 2)$ and $(13, 5)$

$m = \frac{5-2}{13-10} = \frac{3}{3} = 1$

$y = (x-10)+2$
or
$y = (x-13)+5$

It is known that y is an exponential function and that it passes through the given points. Write an equation for this function.

20. $(1, 4)$ and $(3, 10)$

$y = a \cdot r^x$

$10 = 4 \cdot r^2$

$\frac{5}{2} = r^2$

$r = \sqrt{\frac{5}{2}}$

$y = 4\left(\sqrt{\frac{5}{2}}\right)^{x-1}$
or
$y = 10\left(\sqrt{\frac{5}{2}}\right)^{x-3}$

21. $(4, 15)$ and $(10, 3)$

$3 = 15r^6$

$\frac{1}{5} = r^6$

$r = \sqrt[6]{\frac{1}{5}}$

$y = 15\left(\sqrt[6]{\frac{1}{5}}\right)^{x-4}$
or
$y = 3\left(\sqrt[6]{\frac{1}{5}}\right)^{x-10}$

22. $(10, 2)$ and $(13, 5)$

$5 = 2r^3$

$\frac{5}{2} = r^3$

$r = \sqrt[3]{\frac{5}{2}}$

$y = 2\left(\sqrt[3]{\frac{5}{2}}\right)^{x-10}$
or
$y = 5\left(\sqrt[3]{\frac{5}{2}}\right)^{x-13}$

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2.2 Test Prep

23. Mr. Brust has been collecting He-Man figures for 40 years. The number of figures he owns can be modeled by an arithmetic sequence, where the first year is year 1. The number of figures in year 5 was 12, and the number of figures in year 20 was 167. How many He-Man figures did he have in year 14?

$d = \frac{167-12}{20-5} = \frac{155}{15} = \frac{31}{3}$

$a_n = \frac{31}{3}(n-5)+12$

$a_{14} = \frac{31}{3}(14-5)+12$

$a_{14} = \frac{31}{3}(9)+12$

$= 31(3)+12$

$= 93+12 = \boxed{105}$

24. **Calculator active.** Mr. Bean has lots of siblings and lots of nephews and nieces. The number of people in his family can be modeled using a geometric sequence, where the first generation is generation 1. The number of people after generation 2 is 52. The number of people after generation 4 is 468. How many people will be in the Bean family after generation 6?

$468 = 52r^2$

$9 = r^2$

$3 = r$

$g_n = 52(3)^{n-2}$

$g_6 = 52(3)^{6-2} = \boxed{4{,}212 \text{ people}}$

25. The third term of a sequence is 5, and the fifth term of the sequence is 20. Of the following, which statement is true?

(A) If the sequence is arithmetic, the first term could be -25.

(B) If the sequence is arithmetic, the fourth term could be 7.5.

(C) If the sequence is geometric, the fourth term could be 7.

(D) If the sequence is geometric, the sixth term could be 40.

Arithmetic

$d = 7.5$

$a_1 = -10$

$a_4 = 12.5$

Geometric

$r^2 = \frac{20}{5}$

$r = 2$

$g_4 = 10$

$g_6 = 40$ ✓