2.4 Exponential Function Manipulation

AP Precalculus

Solutions

2.4 Practice

Let $f(x)$ be a function on which a transformation occurs. Let $g(x)$ be a transformation of f. For each problem, name the transformation(s) of f.

1. $f(x) = 3^x$ and $g(x) = f(x) \cdot 27$

$3^x \cdot 3^3$

3^{x+3}

$\boxed{\text{Shift left 3 units.}}$

2. $f(x) = 9^x$ and $g(x) = 3^x$

$(9^{\frac{1}{2}})^x$

$9^{\frac{1}{2}x}$

$\boxed{\text{Horizontal dilation.}}$

3. $f(x) = 7^x$ and $g(x) = \frac{1}{7^x}$

7^{-x}

$\boxed{\text{Reflection across the } y\text{-axis.}}$

4. $f(x) = 4^x$ and $g(x) = f(x) \cdot 16$

$4^x \cdot 4^2$

4^{x+2}

$\boxed{\text{Shift left 2 units.}}$

5. $f(x) = 4^x$ and $g(x) = \frac{f(x)}{16}$.

$\frac{4^x}{4^2} = 4^x \cdot 4^{-2}$

4^{x-2}

$\boxed{\text{Shift right 2 units.}}$

6. $f(x) = 5^x$ and $g(x) = (f(x))^{-3}$

$(5^x)^{-3}$

5^{-3x}

$\boxed{\text{Horizontal dilation and reflection across the } y\text{-axis.}}$

7. $f(x) = 6^x$ and $g(x) = \frac{f(x)}{36}$

$\frac{6^x}{6^2} = 6^x 6^{-2}$

6^{x-2}

$\boxed{\text{Shift right 2 units.}}$

8. $f(x) = 3^x$ and $g(x) = 27^x$

$(3^3)^x$

3^{3x}

$\boxed{\text{Horizontal dilation}}$

9. $f(x) = 4^x$ and $g(x) = -(16)^x$

$-(4^2)^x$

-4^{2x}

$\boxed{\text{Horizontal dilation and reflection across the } x\text{-axis.}}$

10. $f(x) = 16^x$ and $g(x) = 2^x$

$\sqrt[4]{16} = 2$

$(16^{\frac{1}{4}})^x$

$16^{\frac{1}{4}x}$

$\boxed{\text{Horizontal dilation}}$

11. $f(x) = 7^x$ and $g(x) = -\frac{f(x)}{49}$

$-\frac{7^x}{7^2}$

-7^{x-2}

$\boxed{\text{Reflection across the } x\text{-axis and shift right 2.}}$

12. $f(x) = 2^x$ and $g(x) = \frac{f(x)}{16}$

$\frac{2^x}{2^4}$

2^{x-4}

$\boxed{\text{Shift right 4 units.}}$

13. $f(x) = 3^x$ and $g(x) = -3f(x)$

$-3^1 \cdot 3^x$

-3^{x+1}

$\boxed{\text{Reflection across the } x\text{-axis and shift left 1.}}$

14. $f(x) = 2^x$ and $g(x) = f(x) \cdot 32$

$2^x \cdot 2^5$

2^{x+5}

$\boxed{\text{Shift left 5 units.}}$

Evaluate the function at the given input values.

15. Let $h(x) = 2 \cdot 3^{x/2}$. Find $h(1)$

$h(1) = 2 \cdot 3^{\frac{1}{2}}$

$\boxed{2\sqrt{3}}$

16. Let $h(x) = 4 \cdot 4^{x/5}$. Find $h(2)$

$h(2) = 4 \cdot 4^{\frac{2}{5}}$

$\boxed{4\sqrt[5]{16}}$

17. Let $h(x) = 7 \cdot 2^{x/4}$. Find $h(-2)$

$h(-2) = 7 \cdot 2^{-2/4}$

$= 7 \cdot \frac{1}{2^{1/2}}$

$\boxed{\frac{7}{\sqrt{2}}}$

18. Let $h(x) = 2 \cdot 6^{x/3}$. Find $h(-1)$

$h(-1) = 2 \cdot 6^{-1/3}$

$2 \cdot \frac{1}{6^{1/3}}$

$\boxed{2/\sqrt[3]{6}}$

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2.4 Test Prep

19. **Calculator active.** A lake in the Cascade Mountains has frozen over during the winter. As spring brings warmer weather, the ice sheet begins to melt. The table below gives the area of the ice, is square feet, at various times, in days since the beginning of spring.

Time (days)	0	1	2
Area (millions of square feet)	2.5	2	1.6

The area of the ice can be modeled by the function $A(t) = ab^t$, where $A(t)$ is the area, in square feet, at time t days since the beginning of spring.

a. Use the given data to write two equations that can be used to find the values for constants a and b in the expression $A(t)$.

Possible equations are
$\boldsymbol{ab^0 = 2.5}$
$\boldsymbol{ab^1 = 2}$
$\boldsymbol{ab^2 = 1.6}$

b. Find the values for a and b, then write the expression for $A(t)$.

$\boldsymbol{a = 2.5}$

$\boldsymbol{2.5b = 2}$
$\boldsymbol{b = 0.8}$

$\boldsymbol{A(t) = 2.5(0.8)^t}$

c. Use the given data to find the average rate of change of the area from $t = 0$ to $t = 2$ days. Show the computations that lead to your answer.

$\boldsymbol{\frac{A(2)-A(0)}{2-0} = \frac{1.6-2.5}{2} = -0.45}$ **million square feet per day** or $\boldsymbol{-450,000}$ **square feet per day**

d. Interpret the meaning of your answer from part c in the context of the problem.

On average, as the time increases from 0 to 2 days, the area of the ice sheet decreases by 450,000 square feet per day.

e. Use the average rate of change found in part c to estimate the area of the ice, in square feet, at time $t = 3$ days. Show the work that leads to your answer.

$\boldsymbol{y = y_1 + m(x - x_1)}$
$\boldsymbol{y = A(0) - 0.45(t - 0)}$ **or** $\boldsymbol{y = A(2) - 0.45(t - 2)}$

For $\boldsymbol{t = 3}$**, either equation gives the same answer.**
$\boldsymbol{y = 2.5 - 0.45(3) = 1.15}$

The area of the ice at time $\boldsymbol{t = 3}$ **days is** $\boldsymbol{1.15}$ **million square feet.**

f. Does the model found in Part b demonstrate exponential growth or exponential decay? Give a reason for your answer.

Because $\boldsymbol{a > 0}$ **and** $\boldsymbol{0 < b < 1}$ **in the model** $\boldsymbol{A(t)}$**, the model demonstrates exponential decay.**