### 2.5.A Exponential Function Context and Data Modeling

2.5.A Notes

Recall: Exponential functions model growth patterns with successive output values over equal-length input-value intervals are proportional.

Sometimes, looking at a data set you will not see a proportional growth pattern even though it is an exponential function. Look at the following table of values.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | -2 | 1 | 7 | 19 | 43 |

It's hard to see the growth pattern for these output values. But if we add 5 to the output values, it becomes more obvious.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 3 | 6 | 12 | 24 | 48 |

1. Below is a table of values for an exponential function in the form $f(x)=a(b)^{x}+k$. Write the equation for this exponential function.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 6 | 12 | 36 | 132 | 516 |

## The number $e$

$$
e \approx 2.718
$$

The natural base $e$ is often used as the base in exponential functions that model contextual scenarios.

## Exponential Regression

2. The table gives the amount Mr. Kelly owes Mr. Sullivan each year after he loses in fantasy football. No payments have been made, so Mr. Kelly's debt grows each year with interest being applied.

| Year <br> $(t)$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Debt $(y)$ <br> (dollars) | 12 | 20 | 59 | 109 | 267 |

a. Use an exponential regression $y=a b^{x}$ to model these data. Round to three decimals but store the original equation in your calculator.
b. Use the model (stored in your calculator) to predict how much Mr. Kelly will owe Mr. Sullivan in Year 7, assuming he continues to lose.

### 2.5.A Exponential Function Context and Data Modeling

### 2.5.A Practice

## AP Precalculus

Below is a table of values for exponential functions in the form $f(x)=a(b)^{x}+\boldsymbol{k}$. Write the equation that represents each table.
1.

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 2 | 10 | 34 | 106 | 322 |

2. 

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 8 | 13 | 23 | 43 | 83 |

3. 

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 5 | 29 | 149 | 749 | 3,749 |

4. The table presents values of the function $f$ for selected values of $x$.

| $x$ | -8 | -7 | -1 | 2 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 30 | 40 | 158 | 280 |

a. Find an exponential regression $y=a b^{x}$ to model these data. Round to three decimals but store the original equation in your calculator.
b. Use the model stored in your calculator to predict the value of $f(0.5)$.
c. According to the model, when will $f(x)=100$ ?
d. According to the model, when will $f(x)=1,000$ ?
5. The table gives the number of cells $c$ of a culture of bacteria found in a petri dish after $t$ days.

| Days <br> $(t)$ | 3 | 5 | 10 | 13 |
| :---: | :---: | :---: | :---: | :---: |
| Cells <br> $(c)$ | 300 | 350 | 966 | 1,807 |

a. Use an exponential regression $c(t)=a b^{t}$ to model these data. Round to three decimals but store the original equation in your calculator.
b. According to the model, how many cells were in the culture of bacteria after 8 days? Use the model stored in your calculator, not the rounded answer from part a.
c. Use the model stored in your calculator to predict the number of bacteria cells after 31 days.
d. How many days will it take for the culture to have 1 million cells?
6. The table gives the population of a city, in millions, for selected years where $t=2$ represents 2012 and $t=5$ represents 2015.

| Year | 2012 | 2015 | 2018 | 2021 |
| :---: | :---: | :---: | :---: | :---: |
| Population <br> (millions) | 0.801 | 0.933 | 1.36 | 1.85 |

a. Use an exponential regression $f(t)=a b^{t}$ to model these data. Round to three decimals but store the original equation in your calculator.
b. Use the model stored in your calculator to predict the population of the city in 2030.
c. During what year did the population reach 1.5 million?
d. If the model holds, during what year will the population reach 3 million?
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7. Mr. Bean had a caffeinated soda at 8:00 a.m. The amount of caffeine (measured in mg) in Mr. Bean's system $t$ hours is shown in the table below.

| Hours <br> $(t)$ | 1 | 3 | 8 | 12 | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Caffeine <br> $(\mathrm{mg})$ | 128 | 85 | 47 | 20 | 18 |

a. Use an exponential regression $C(t)=a b^{t}$ to model these data. Round to three decimals but store the original equation in your calculator.
b. Use the model stored in your calculator to predict the amount of caffeine still in Mr. Bean's body after 15 hours.
c. How long will it take before the caffeine level is 5 mg ?
d. How long will it take before the caffeine level is 1 mg ?

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8. Calculator active. In the $x y$-plane, the graphs of the linear function $L$ and the exponential function $E$ both pass through the points $(0,4)$ and $(1,8)$. The exponential function $E$ is in the form $E(x)=a b^{x}$. The function $f$ is given by $f(x)=E(x)-L(x)$. What is the minimum value of $f$.
