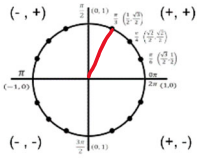
3.11 Secant, Cosecant, and Cotangent Functions

AP Precalculus

3.11 Practice

Evaluate the following expressions. Use exact values.

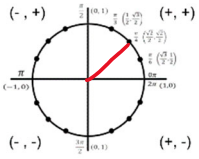
1. $\sec\left(\frac{\pi}{3}\right)$



$\frac{1}{\cos\left(\frac{\pi}{3}\right)}$

$\frac{1}{\frac{1}{2}} = 1 \cdot \frac{2}{1} = \boxed{2}$

2. $\csc\left(\frac{\pi}{4}\right)$

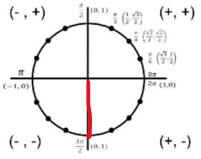


$\frac{1}{\sin\left(\frac{\pi}{4}\right)}$

$\frac{1}{\frac{\sqrt{2}}{2}} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{2}}{2}$

$\boxed{\sqrt{2}}$

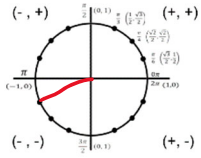
3. $\cot\left(\frac{3\pi}{2}\right)$



$\frac{1}{\tan\left(\frac{3\pi}{2}\right)}$

$\frac{1}{\frac{1}{0}} = 1 \cdot \frac{0}{1} = \frac{0}{1} = \boxed{0}$

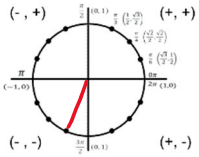
4. $\sec\left(\frac{7\pi}{6}\right)$



$\frac{1}{\cos\left(\frac{7\pi}{6}\right)}$

$\frac{1}{-\frac{\sqrt{3}}{2}} = 1 \cdot \left(-\frac{2}{\sqrt{3}}\right) = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

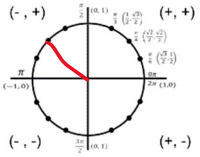
5. $\csc\left(-\frac{2\pi}{3}\right)$



$\frac{1}{\sin\left(\frac{4\pi}{3}\right)}$

$\frac{1}{-\frac{\sqrt{3}}{2}} = 1 \cdot \left(-\frac{2}{\sqrt{3}}\right) = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

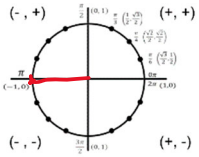
6. $\cot\left(\frac{3\pi}{4}\right)$



$\frac{1}{\tan\left(\frac{3\pi}{4}\right)}$

$\frac{1}{-\frac{\sqrt{2}}{\sqrt{2}}} = \frac{1}{-1} = \boxed{-1}$

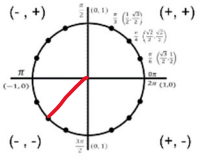
7. $\csc(\pi)$



$\frac{1}{\sin\pi}$

$\frac{1}{0} = \boxed{\text{Undefined}}$

8. $\sec\left(\frac{5\pi}{4}\right)$

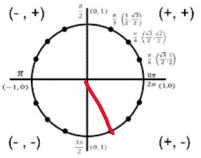


$\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\frac{1}{-\frac{\sqrt{2}}{2}} = 1 \cdot \left(-\frac{2}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2}$

$\boxed{-\sqrt{2}}$

9. $\cot\left(\frac{5\pi}{3}\right)$



$\frac{1}{\tan\left(\frac{5\pi}{3}\right)}$

$\frac{1}{-\frac{\sqrt{3}}{1}} = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

Evaluate the following expressions. Use approximate values from calculator.

10. $\csc(1.43)$

$\frac{1}{\sin(1.43)} \approx 1.009$

11. $\sec\left(\frac{\pi}{5}\right)$

$\frac{1}{\cos\left(\frac{\pi}{5}\right)} \approx 1.236$

12. $\cot\left(\frac{5\pi}{7}\right)$

$\frac{1}{\tan\left(\frac{5\pi}{7}\right)} \approx -0.797$

13. $\sec(1.3\pi)$

$\frac{1}{\cos(1.3\pi)} \approx -1.701$

14. $\cot(-3.26)$

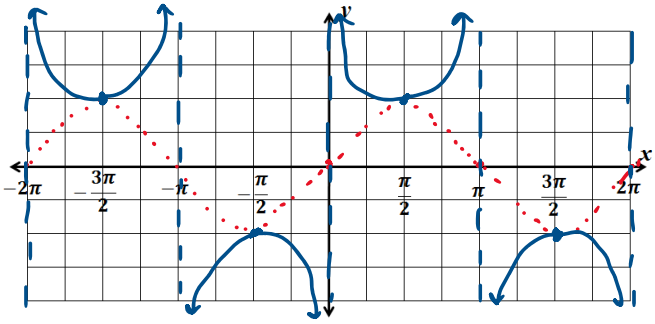
$\frac{1}{\tan(-3.26)} \approx -8.405$

15. $\csc\left(\frac{\pi}{9}\right)$

$\frac{1}{\sin\left(\frac{\pi}{9}\right)} \approx 2.923$

Graph the following and state all vertical asymptotes.

16. $f(x) = 2\csc x$



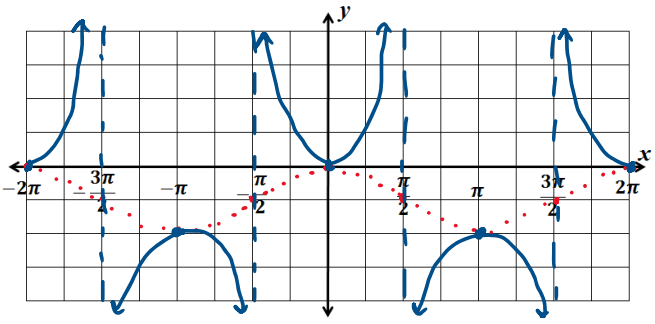
Range:

$(-\infty, -2] \cup [2, \infty)$

Vertical Asymptotes:

$x = 0 + \pi n$

17. $f(x) = \sec(x) - 1$



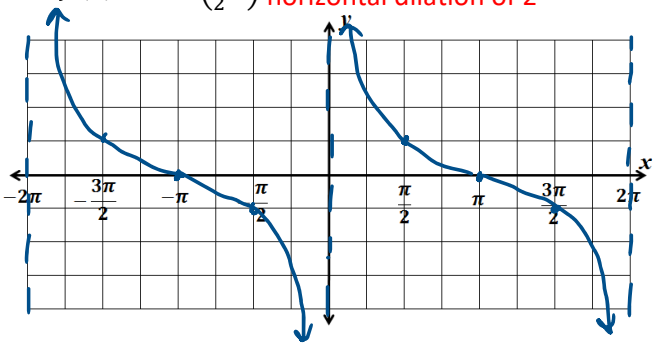
Range:

$(-\infty, -2] \cup [0, \infty)$

Vertical Asymptotes:

$x = \frac{\pi}{2} + \pi n$

18. $f(x) = \cot\left(\frac{1}{2}x\right)$ cotangent is undefined at 0, π, 2π… horizontal dilation of 2



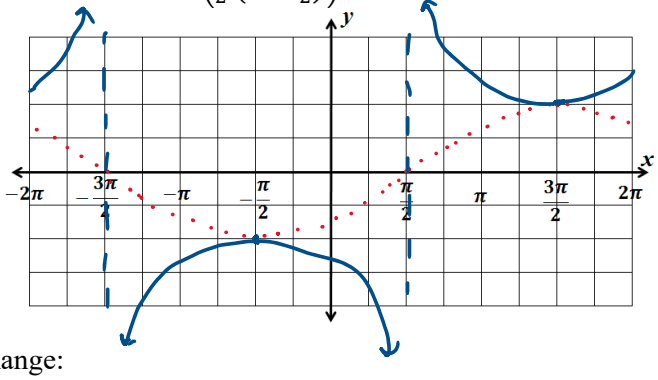
Range:

$(-\infty, \infty)$

Vertical Asymptotes:

$x = 0 + 2\pi n$

19. $f(x) = 2\csc\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$



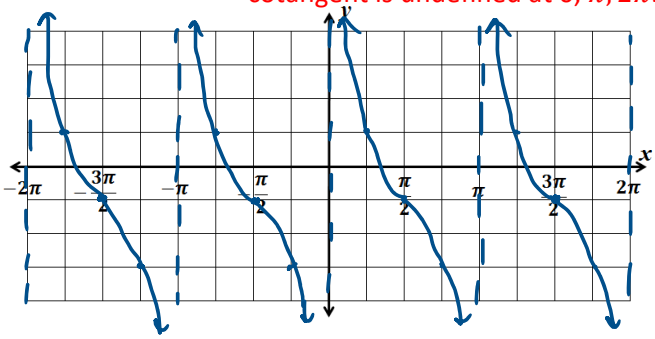
Range:

$(-\infty, -2] \cup [2, \infty)$

Vertical Asymptotes:

$x = \frac{\pi}{2} + 2\pi n$

20. $f(x) = 2\cot(x) - 1$ cotangent is undefined at 0, π, 2π…



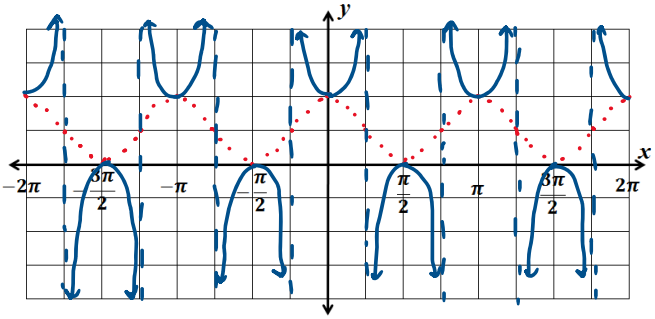
Range:

$(-\infty, \infty)$

Vertical Asymptotes:

$x = 0 + \pi n$

21. $f(x) = \sec(2x) + 1$



Range:

$(-\infty, 0] \cup [2, \infty)$

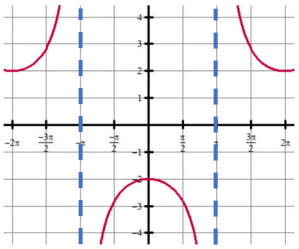
Vertical Asymptotes:

$x = \frac{\pi}{4} + \frac{\pi}{2} n$

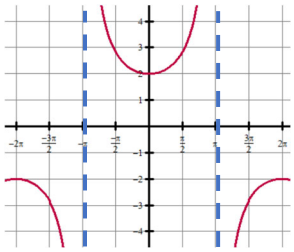
3.11 Test Prep

21. Which of the following is the graph of $f(x) = -2\csc\left(\frac{1}{2}x\right)$?

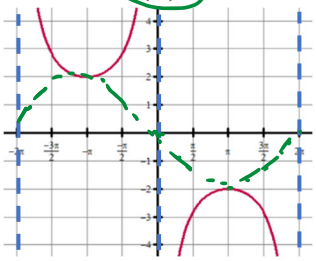
(A)



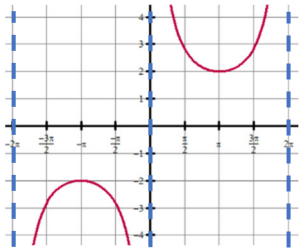
(B)



(C)

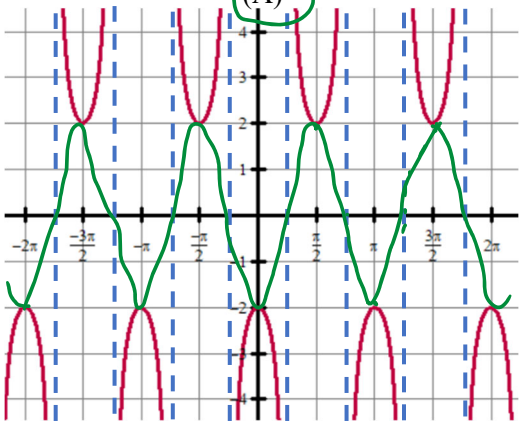


(D)

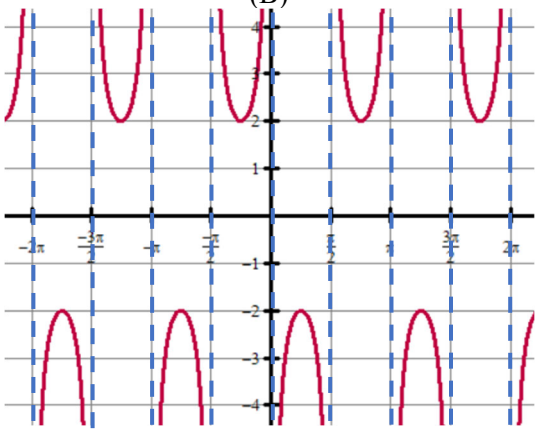


22. Which of the following is the graph of $f(\theta) = 2\sec\left(2\left(\theta + \frac{\pi}{2}\right)\right)$?

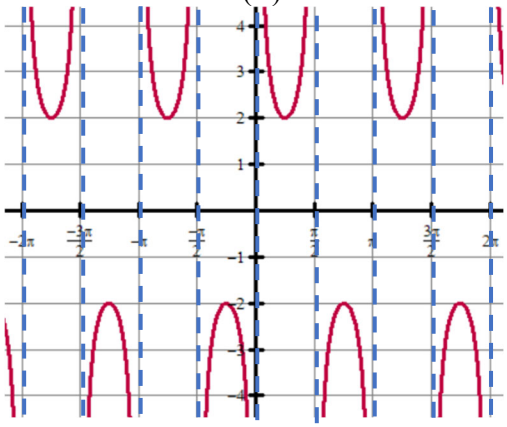
(A)



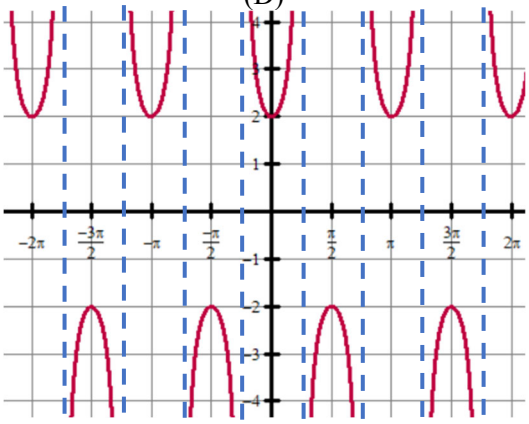
(B)



(C)



(D)



23. Which of the following describes the graph of $f(x) = 2\cot x$? cotangent is undefined at 0, π, 2π…

(A) Vertical asymptotes at $x = \frac{\pi}{2} + \pi k$, where k is an integer, and the range is all real numbers.

(B) Vertical asymptotes at $x = \frac{\pi}{2} + \pi k$, where k is an integer, and the range is $(-\infty, -2] \cup [2, \infty)$.

(C) Vertical asymptotes at $x = \pi + \pi k$, where k is an integer, and the range is all real numbers.

(D) Vertical asymptotes at $x = \pi + \pi k$, where k is an integer, and the range is $(-\infty, -2] \cup [2, \infty)$.