3.12A Equivalent Representations of Trig Functions

AP Precalculus

3.12A Practice

Use trig identities to write each expression in terms of a single trig identity.

1. $\cot x \sin x$

$\frac{\cos x}{\sin x} \cdot \sin x$

$\cos x$

2. $\frac{1-\sin^2 x}{\sin^2 x}$

$\frac{\cos^2 x}{\sin^2 x}$

$\cot^2 x$

3. $\sin^2 x + \cos^2 x + \tan^2 x$

$1 + \tan^2 x$

$\sec^2 x$

4. $\sin\theta \sec\theta$

$\sin\theta \cdot \frac{1}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}$

$\tan\theta$

5. $\frac{\sec^2 x-1}{\sin^2 x}$

$\frac{\tan^2 x}{\sin^2 x}$

$\tan^2 x \cdot \left(\frac{1}{\sin^2 x}\right)$

$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x}$

$\frac{1}{\cos^2 x}$

$\sec^2 x$

6. $\cot^2\theta\,(1-\cos^2\theta)$

$\cot^2\theta(\sin^2\theta)$

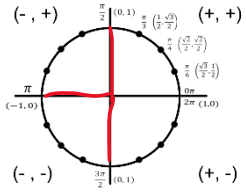
$\frac{\cos^2\theta}{\sin^2\theta} \cdot \sin^2\theta$

$\cos^2\theta$

$\sin^2 + \cos^2 = 1$

Use trig identities to solve the trig equations for $0 \le x \le 2\pi$. Find exact values.

7. $\sin^2 x - \cos x = 1$



$-1 \quad -1$

$\sin^2 x - 1 - \cos x = 0$

$-\cos^2 x - \cos x = 0$

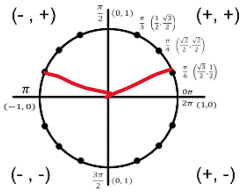
$-\cos x(\cos x + 1) = 0$

$\cos x = 0 \quad | \quad \cos x = -1$

$\frac{\pi}{2}, \frac{3\pi}{2} \qquad \pi$

$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

8. $2\cos x \tan x = 1$



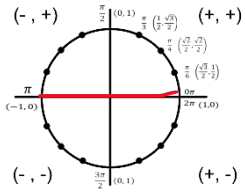
$2\cos x \cdot \frac{\sin x}{\cos x} = 1$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

9. $\sec^2 x + \tan^2 x = 1$



$1 + \tan^2 x + \tan^2 x = 1$

$1 + 2\tan^2 x = 1$

$-1 \qquad -1$

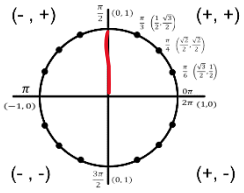
$\frac{2\tan^2 x}{2} = \frac{0}{2}$

$\sqrt{\tan^2 x} = \sqrt{0}$

$\tan x = 0$

$x = 0, \pi, 2\pi$

10. $\sin^3 x\,(1 + \cot^2 x) = 1$



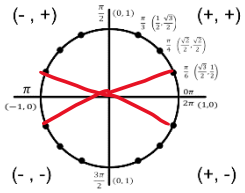
$\sin^3 x(\csc^2 x) = 1$

$\sin x \cdot \frac{\sin^2 x}{\sin^2 x} \cdot \frac{1}{1} = 1$

$\sin x = 1$

$x = \frac{\pi}{2}$

11. $\frac{\cos x}{\sec x} = \frac{3}{4}$



$\frac{\cos x}{\frac{1}{\cos x}} = \frac{3}{4}$

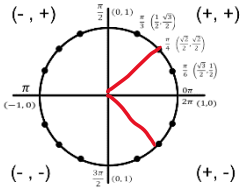
$\cos x\left(\frac{\cos x}{1}\right) = \frac{3}{4}$

$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

12. $(1 - \sin^2 x)\sec x = \frac{\sqrt{2}}{2}$



$\cos^2 x \cdot \frac{1}{\cos x} = \frac{\sqrt{2}}{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

13. The function f is given by $f(x) = \frac{\csc^2 x}{\sec^2 x}$. Which of the following expressions is equivalent to $f(x)$?

(A) $\cos^2 x$

(B) $\tan^2 x$

(C) $\sin^2 x$

(D) $\cot^2 x$

$$\frac{\frac{1}{\sin^2 x}}{\frac{1}{\cos^2 x}} = \frac{1}{\sin^2 x} \cdot \frac{\cos^2 x}{1} = \frac{\cos^2 x}{\sin^2 x}$$

14. The function f is given by $f(\theta) = \frac{\csc\theta \cot\theta \sin\theta}{\cos\theta}$. Which of the following expressions is equivalent to $f(\theta)$?

(A) $\csc\theta$

(B) $\sin\theta$

(C) $\sec\theta$

(D) $\tan\theta$

$$\frac{\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta} \cdot \sin\theta}{\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta} = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} = \frac{1}{\sin\theta}$$

15. The function g is defined by $g(x) = \csc^2 x + \cot x$. What are all solutions to $g(x) = 1$ on the interval $0 \le x \le 2\pi$?

(A) $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$

(B) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{4}$

(C) $x = \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

(D) $x = \frac{\pi}{2}$

$$1 = \csc^2 x + \cot x$$

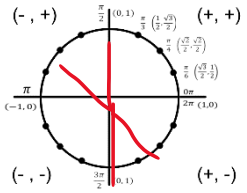
$$1 = \cot^2 x + 1 + \cot x$$

$$-1 \qquad -1$$

$$0 = \cot^2 x + \cot x$$

$$0 = \cot x(\cot x + 1)$$

$\cot x = 0$ $\quad$ $\cot x = -1$



16. The function g is given by $g(x) = \frac{1-\cos^2 x}{\cos^2 x} + 1$. Which of the following expressions is equivalent to $g(x)$?

(A) $\cot^2 x$

(B) $\tan^2 x$

(C) $\csc^2 x$

(D) $\sec^2 x$

$$= \frac{\sin^2 x}{\cos^2 x} + 1$$

$$= \tan^2 x + 1$$

$$= \sec^2 x$$