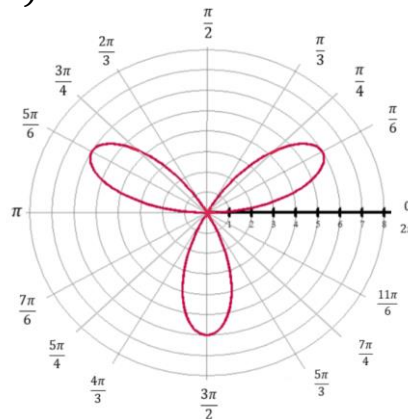
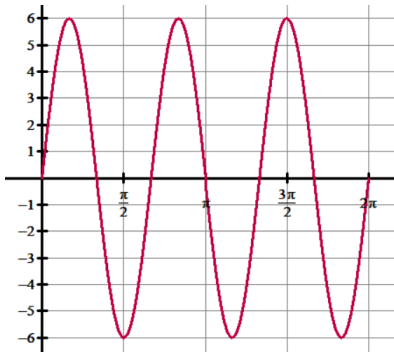


Write your questions  
and thoughts here!Rate of Change of  $r$ **EXAMPLE #1:**  $r = 6\sin(3\theta)$  $r$  on the intervals:

$\left(0, \frac{\pi}{6}\right)$

$\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

$\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

## Distance from the Pole

	$r$ is positive	$r$ is negative
$r = f(\theta)$ is increasing		
$r = f(\theta)$ is decreasing		

**EXAMPLE #2:**  $r = f(\theta)$ 

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$
$r$	0	2.5	4.33	5	4.33	2.5	0

- Describe the behavior of the function on the interval  $0 \leq \theta \leq \frac{\pi}{2}$ .
- Describe the behavior of the function on the interval  $\frac{\pi}{2} \leq \theta \leq \pi$ .
- What is the average rate of change on the interval  $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$ ?
- Estimate the value of  $f\left(\frac{\pi}{4}\right)$ .

**EXAMPLE #3:**  $r = f(\theta) = 3 + 5\sin(\theta)$  on the interval  $\pi \leq \theta \leq 2\pi$

$\theta$	$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
$r$									

- Determine the interval(s) where  $f$  is increasing.
- Determine the interval(s) where  $f$  is decreasing.
- Is there at least one relative extrema on the interval  $\pi \leq \theta \leq 2\pi$ ? Explain.
- The distance between  $f(\theta)$  and the pole is increasing or decreasing on the interval  $\frac{5\pi}{4} \leq \theta \leq \frac{3\pi}{2}$ . Justify your answer.
- The distance between  $f(\theta)$  and the pole is increasing or decreasing on the interval  $\pi \leq \theta \leq \frac{7\pi}{6}$ . Justify your answer.
- Find the average rate of change of  $f$  between  $\theta = \frac{3\pi}{2}$  and  $\theta = \frac{7\pi}{4}$ .
- On what interval(s) must  $f(\theta) = 0$ .
- Compare the estimated value  $f\left(\frac{5\pi}{3}\right)$  to the real value of  $f\left(\frac{5\pi}{3}\right)$ .

### 3.15 Rates of Change in Polar Functions

### 3.15 Practice

AP Precalculus

Use the table of selected values for the polar function  $r = f(\theta)$  to answer the following.

1.

$\theta$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	$2\pi$
$r$	11	9.53	6	2.46	1	2.46	6	9.53	11

- a. Determine the interval(s) where  $f$  is increasing. Determine the interval(s) where  $f$  is decreasing.
- b. The distance between  $f(\theta)$  and the pole is increasing or decreasing on the interval  $\pi \leq \theta \leq 2\pi$ . Justify your answer.
- c. Find the average rate of change of  $f$  between  $\theta = \frac{5\pi}{4}$  and  $\theta = \frac{7\pi}{4}$ .
- d. Estimate the value of  $f\left(\frac{\pi}{3}\right)$  using an average rate of change.
- e. Are there any extrema on the interval  $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ ? Explain how you know.

- 2.
- a. Is  $f$  increasing or decreasing on the interval  $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$ ?
- b. Is the distance between  $f(\theta)$  and the pole is increasing or decreasing on the interval  $0 \leq \theta \leq \frac{\pi}{4}$ ?
- c. Is the rate of change of  $f$  faster on the interval  $\left[0, \frac{\pi}{8}\right]$  or the interval  $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ ? Justify.

$\theta$	$r$
0	0
$\frac{\pi}{8}$	-1.41
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	-1.41
$\frac{\pi}{2}$	0

Use the polar function  $r = f(\theta)$  to fill in the table and answer the questions. Calculator Active.

3.  $r = f(\theta) = 8 \cos(\theta)$

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	$2\pi$
$r$									

- Determine the interval(s) where  $f$  is increasing. Determine the interval(s) where  $f$  is decreasing.
- How many extrema on the interval  $\frac{5\pi}{6} \leq \theta \leq \frac{11\pi}{6}$  ?
- Determine the intervals where the distance between  $f(\theta)$  and the pole is increasing on the interval  $0 \leq \theta \leq 2\pi$  . Justify your answer.
- Determine the intervals where the distance between  $f(\theta)$  and the pole is decreasing on the interval  $0 \leq \theta \leq 2\pi$  . Justify your answer.
- Find the average rate of change of  $f$  between  $\theta = \frac{5\pi}{6}$  and  $\theta = \pi$  . Use to estimate  $f\left(\frac{2\pi}{3}\right)$ .

4.  $r = f(\theta) = -3 + 5 \sin(\theta)$

- Is the distance between  $f(\theta)$  and the pole is increasing or decreasing on the interval  $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$  ?
- Find the average rate of change of  $f$  between  $\theta = \frac{\pi}{4}$  and  $\theta = \frac{\pi}{2}$  .
- Estimate the value of  $f\left(\frac{5\pi}{6}\right)$  using an average rate of change.

$\theta$	$r$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
$\pi$	

5. Consider the graph of the polar function  $r = f(\theta)$ , where  $\theta$  is increasing in the polar coordinate system on the interval  $0 \leq \theta \leq 2\pi$ . Given  $f(\theta) < 0$  and decreasing on the interval  $\pi \leq \theta \leq \frac{3\pi}{2}$  which of the following statements is true about the distance between the point with polar coordinates  $(f(\theta), \theta)$  and the origin.
- (A) The distance is increasing for  $0 \leq \theta \leq 2\pi$ .
- (B) The distance is decreasing for  $0 \leq \theta \leq 2\pi$ .
- (C) The distance is increasing for  $\pi \leq \theta \leq \frac{3\pi}{2}$ .
- (D) The distance is decreasing for  $\pi \leq \theta \leq \frac{3\pi}{2}$ .

Use the table of selected values for the polar equation  $r = f(\theta)$  below to answer questions 6 and 7.

**CALCULATOR ACTIVE**

$\theta$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$r$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

6. The graph of the polar function  $r = f(\theta)$ , is given the polar coordinate system. Which of the following descriptions is true ?
- (A) As  $\theta$  increasing from 0 to  $\frac{\pi}{4}$ , the polar function  $r = f(\theta)$  is increasing, and the distance between the point  $(f(\theta), \theta)$  on the curve and the origin is increasing.
- (B) As  $\theta$  increasing from 0 to  $\frac{\pi}{4}$ , the polar function  $r = f(\theta)$  is increasing, and the distance between the point  $(f(\theta), \theta)$  on the curve and the origin is decreasing.
- (C) As  $\theta$  increasing from 0 to  $\frac{\pi}{4}$ , the polar function  $r = f(\theta)$  is decreasing, and the distance between the point  $(f(\theta), \theta)$  on the curve and the origin is increasing.
- (D) As  $\theta$  increasing from 0 to  $\frac{\pi}{4}$ , the polar function  $r = f(\theta)$  is decreasing, and the distance between the point  $(f(\theta), \theta)$  on the curve and the origin is decreasing.
7. If the value of  $r = f\left(\frac{\pi}{8}\right)$  is estimated using the average rate of change of the function over the interval  $0 \leq \theta \leq 2\pi$ , which of the following is true?
- (A) The estimated value would be an overestimate of the actual value by approximately 0.116.
- (B) The estimated value would be an underestimate of the actual value by approximately -0.23.
- (C) The estimated value would be an overestimate of the actual value by approximately 1.616.
- (D) The estimated value would be an underestimate of the actual value by approximately -1.043.