AP Precalc

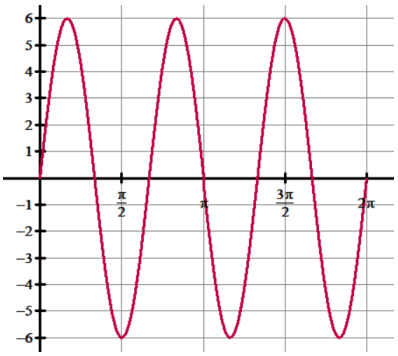
Write your questions and thoughts here!

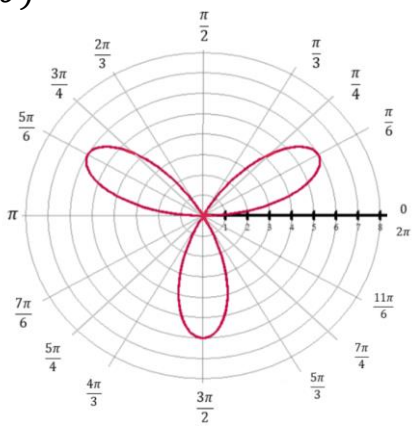
3.15 Rates of Change in Polar Functions

3.15 Notes

Rate of Change of r

EXAMPLE #1: $r = 6\sin(3\theta)$





r on the intervals:

$\left(\mathbf{0}, \frac{\pi}{6}\right)$

$\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

$\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

Distance from the Pole

	r is positive	r is negative
$r = f(\theta)$ is increasing		
$r = f(\theta)$ is decreasing		

EXAMPLE #2: $r = f(\theta)$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	0	2.5	4.33	5	4.33	2.5	0

a. Describe the behavior of the function on the interval $0 \le \theta \le \frac{\pi}{2}$.

b. Describe the behavior of the function on the interval $\frac{\pi}{2} \le \theta \le \pi$.

c. What is the average rate of change on the interval $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$?

d. Estimate the value of $f\left(\frac{\pi}{4}\right)$.

Write your questions and thoughts here!

EXAMPLE #3: $r = f(\theta) = 3 + 5\sin(\theta)$ on the interval $\pi \leq \theta \leq 2\pi$

$\boldsymbol{\theta}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\boldsymbol{r}$									

a. Determine the interval(s) where f is increasing.

b. Determine the interval(s) where f is decreasing.

c. Is there at least one relative extrema on the interval $\pi \leq \theta \leq 2\pi$? Explain.

d. The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\frac{5\pi}{4} \leq \theta \leq \frac{3\pi}{2}$. Justify your answer.

e. The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\pi \leq \theta \leq \frac{7\pi}{6}$. Justify your answer.

f. Find the average rate of change of f between $\theta = \frac{3\pi}{2}$ and $\theta = \frac{7\pi}{4}$.

g. On what interval(s) must $f(\theta) = 0$.

h. Compare the estimated value $f\left(\frac{5\pi}{3}\right)$ to the real value of $f\left(\frac{5\pi}{3}\right)$.

3.15 Rates of Change in Polar Functions

AP Precalculus

3.15 Practice

Use the table of selected values for the polar function $r = f(\theta)$ to answer the following.

1.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	11	9.53	6	2.46	1	2.46	6	9.53	11

a. Determine the interval(s) where f is increasing. Determine the interval(s) where f is decreasing.

b. The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\pi \le \theta \le 2\pi$. Justify your answer.

c. Find the average rate of change of f between $\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$.

d. Estimate the value of $f\left(\frac{\pi}{3}\right)$ using an average rate of change.

e. Are there any extrema on the interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$? Explain how you know.

2.

a. Is f increasing or decreasing on the interval $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$?

b. Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $0 \le \theta \le \frac{\pi}{4}$?

c. Is the rate of change of f faster on the interval $\left[0, \frac{\pi}{8}\right]$ or the interval $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$? Justify.

θ	r
0	0
$\frac{\pi}{8}$	-1.41
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	-1.41
$\frac{\pi}{2}$	0

Use the polar function $r = f(\theta)$ to fill in the table and answer the questions. Calculator Active.

3. $r = f(\theta) = 8\cos(\theta)$

$\boldsymbol{\theta}$	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\boldsymbol{r}$									

a. Determine the interval(s) where f is increasing. Determine the interval(s) where f is decreasing.

b. How many extrema on the interval $\frac{5\pi}{6} \le \theta \le \frac{11\pi}{6}$?

c. Determine the intervals where the distance between $f(\theta)$ and the pole is increasing on the interval $0 \le \theta \le 2\pi$. Justify your answer.

d. Determine the intervals where the distance between $f(\theta)$ and the pole is decreasing on the interval $0 \le \theta \le 2\pi$. Justify your answer.

e. Find the average rate of change of f between $\theta = \frac{5\pi}{6}$ and $\theta = \pi$. Use to estimate $f\left(\frac{2\pi}{3}\right)$.

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4. $r = f(\theta) = -3 + 5\sin(\theta)$

a. Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$?

b. Find the average rate of change of f between $\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{2}$.

c. Estimate the value of $f\left(\frac{5\pi}{6}\right)$ using an average rate of change.

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	

5. Consider the graph of the polar function $r = f(\theta)$, where θ is increasing in the polar coordinate system on the interval $0 \le \theta \le 2\pi$. Given $f(\theta) < 0$ and decreasing on the interval $\pi \le \theta \le \frac{3\pi}{2}$ which of the following statements is true about the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin.

   (A) The distance is increasing for $0 \le \theta \le 2\pi$.

   (B) The distance is decreasing for $0 \le \theta \le 2\pi$.

   (C) The distance is increasing for $\pi \le \theta \le \frac{3\pi}{2}$.

   (D) The distance is decreasing for $\pi \le \theta \le \frac{3\pi}{2}$.

Use the table of selected values for the polar equation $r = f(\theta)$ below to answer questions 6 and 7.

CALCULATOR ACTIVE

θ	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
r	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

6. The graph of the polar function $r = f(\theta)$, is given the polar coordinate system. Which of the following descriptions is true ?

   (A) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

   (B) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

   (C) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

   (D) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

7. If the value of $r = f\left(\frac{\pi}{8}\right)$ is estimated using the average rate of change of the function over the interval $0 \le \theta \le 2\pi$, which of the following is true?

   (A) The estimated value would be an overestimate of the actual vale by approximately 0.116.

   (B) The estimated value would be an underestimate of the actual vale by approximately -0.23.

   (C) The estimated value would be an overestimate of the actual vale by approximately 1.616.

   (D) The estimated value would be an underestimate of the actual vale by approximately -1.043.