3.15 Rates of Change in Polar Functions

AP Precalculus

3.15 Practice

Use the table of selected values for the polar function $r = f(\theta)$ to answer the following.

1.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	11	9.53	6	2.46	1	2.46	6	9.53	11	

a. Determine the interval(s) where f is increasing. Determine the interval(s) where f is decreasing.

increasing $(\pi, 2\pi)$ decreasing $(0, \pi)$

b. The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\pi \le \theta \le 2\pi$. Justify your answer.

r is positive and increasing so the distance is increasing

c. Find the average rate of change of f between $\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$.

$$\frac{9.53-2.46}{\frac{7\pi}{4}-\frac{5\pi}{4}} = \frac{7.07}{\frac{2\pi}{4}} = \frac{7.07}{\frac{\pi}{2}} = \frac{14.14}{\pi} \approx 4.5 \text{ units per radian}$$

d. Estimate the value of $f\left(\frac{\pi}{3}\right)$ using an average rate of change.

Use the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ $\frac{6-9.53}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-3.53}{\frac{\pi}{4}} = \frac{-14.16}{\pi} \approx -4.495$

$y - y_1 = m(x - x_1)$

$y - 6 = -4.495\left(x - \frac{\pi}{2}\right)$

$y - 6 = -4.495x + 7.06$

$+6 \quad +6$

$y = -4.495x + 13.06$

$y = -4.495\left(\frac{\pi}{3}\right) + 13.06$

$y = 8.352$

Are there any extrema on the interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$? Explain how you know.

yes, there is at least one because the function goes from decreasing to increasing

---

2.

a. Is f increasing or decreasing on the interval $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$?

increasing

b. Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $0 \le \theta \le \frac{\pi}{4}$?

r is negative and decreasing so the distance is increasing

c. Is the rate of change of f faster on the interval $\left[0, \frac{\pi}{8}\right]$ or the interval $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$? Justify.

θ	r
0	0
$\frac{\pi}{8}$	-1.41
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	-1.41
$\frac{\pi}{2}$	0

$$\frac{-1.41-0}{\frac{\pi}{8}-0} = \frac{-1.41}{\frac{\pi}{8}} = \frac{-11.28}{\pi} \approx -3.59$$

$$\frac{-2-(-1.41)}{\frac{\pi}{4}-\frac{\pi}{8}} = \frac{-0.59}{\frac{\pi}{8}} = \frac{-4.72}{\pi} \approx -1.502$$

Faster on $\left[0, \frac{\pi}{8}\right]$ because is -3.59 is more negative (steeper) than -1.502

Use the polar function $r = f(\theta)$ to fill in the table and answer the questions. Calculator Active.

3. $r = f(\theta) = 8\cos(\theta)$ $\frac{2\pi}{3}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
r	8	6.928	0	-6.928	-8	-6.928	0	6.928	8

a. Determine the interval(s) where f is increasing. Determine the interval(s) where f is decreasing.

increasing on $(\pi, 2\pi)$ decreasing on $(0, \pi)$

b. How many extrema on the interval $\frac{5\pi}{6} \le \theta \le \frac{11\pi}{6}$? Justify.

there is at least one because the function goes from decreasing to increasing

c. Determine the intervals where the distance between $f(\theta)$ and the pole is increasing on the interval $0 \le \theta \le 2\pi$. Justify your answer.

increasing on $\left(\frac{\pi}{2}, \pi\right)$ because r is negative and decreasing

increasing on $\left(\frac{3\pi}{2}, \pi\right)$ because r is positive and increasing

d. Determine the intervals where the distance between $f(\theta)$ and the pole is decreasing on the interval $0 \le \theta \le 2\pi$. Justify your answer.

decreasing on $\left(\pi, \frac{3\pi}{2}\right)$ because r is negative and increasing

decreasing on $\left(0, \frac{\pi}{2}\right)$ because r is positive and decreasing

e. Find the average rate of change of f between $\theta = \frac{\pi}{2}$ and $\theta = \frac{5\pi}{6}$. Use to estimate $f\left(\frac{2\pi}{3}\right)$.

$$\frac{0-(-6.928)}{\frac{\pi}{2}-\frac{5\pi}{6}} = \frac{6.928}{-\frac{2\pi}{6}} = \frac{6.928}{-\frac{\pi}{3}} = \frac{6.928}{-\pi} \approx -6.615$$

$y - y_1 = m(x - x_1)$

$y - 0 = -6.615\left(x - \frac{\pi}{2}\right)$

$y = -6.615x + 10.39$

$y = -6.615\left(\frac{2\pi}{3}\right) + 10.39$

$y = -3.464$

$f\left(\frac{2\pi}{3}\right) \approx -3.464$

4. $r = f(\theta) = -3 + 5\sin(\theta)$

a. Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$?

decreasing on $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ because r is positive and decreasing

θ	r
0	-3
$\frac{\pi}{4}$	0.535
$\frac{\pi}{2}$	2
$\frac{3\pi}{4}$	0.535
π	-3

b. Find the average rate of change of f between $\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{2}$.

$$\frac{2-0.535}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{1.465}{\frac{\pi}{4}} = \frac{5.86}{\pi} \approx 1.865 \text{ units per radian}$$

c. Estimate the value of $f\left(\frac{5\pi}{6}\right)$ using an average rate of change

Use the interval $\left[\frac{3\pi}{4}, \pi\right]$ $\quad \frac{0.535-(-3)}{\frac{3\pi}{4}-\pi} = \frac{3.535}{-\frac{\pi}{4}} = \frac{14.14}{-\pi} \approx -4.5$

$y - (-3) = -4.5(x - \pi)$

$y + 3 = -4.5x + 4.5\pi$

$-3 \qquad -3$

$y = -4.5x + 11.137$

$y \quad 4.5\left(\frac{5\pi}{6}\right) + 11.137 = -0.643$

$f\left(\frac{5\pi}{6}\right) \approx -0.643$

3.15 Rates of Change in Polar Functions

3.15 Test Prep

5. Consider the graph of the polar function $r = f(\theta)$, where θ is increasing in the polar coordinate system on the interval $0 \le \theta \le 2\pi$. Given $f(\theta) < 0$ and decreasing on the interval $\pi \le \theta \le \frac{3\pi}{2}$ which of the following statements is true about the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin.

(A) The distance is increasing for $0 \le \theta \le 2\pi$.

(B) The distance is decreasing for $0 \le \theta \le 2\pi$.

(C) The distance is increasing for $\pi \le \theta \le \frac{3\pi}{2}$.

(D) The distance is decreasing for $\pi \le \theta \le \frac{3\pi}{2}$.

because r is negative and decreasing on $\left(\pi, \frac{3\pi}{2}\right)$ so the distance is increasing

Use the table of selected values for the polar equation $r = f(\theta)$ below to answer questions 6 and 7.

CALCULATOR ACTIVE

θ	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
r	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

$-1 \quad -0.866 \quad -0.5$

r is negative and increasing so the distance is decreasing

6. The graph of the polar function $r = f(\theta)$, is given the polar coordinate system. Which of the following descriptions is true ?

(A) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

(B) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

(C) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

(D) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

7. If the value of $r = f\left(\frac{\pi}{8}\right)$ is estimated using the average rate of change of the function over the interval $0 \le \theta \le 2\pi$, which of the following is true?

(A) The estimated value would be an overestimate of the actual value by approximately 0.116.

(B) The estimated value would be an underestimate of the actual value by approximately -0.884.

(C) The estimated value would be an overestimate of the actual value by approximately 1.616.

(D) The estimated value would be an underestimate of the actual value by approximately -1.043.

Use the interval $\left[0, \frac{\pi}{4}\right]$ to estimate

$$\frac{-0.5-(-1)}{\frac{\pi}{4}-0} = \frac{0.5}{\frac{\pi}{4}} = \frac{2}{\pi} \approx 0.636$$

$y - (-1) = \frac{2}{\pi}(x - 0)$

$y + 1 = \frac{2}{\pi}x$

$-1 \qquad -1$

$y = \frac{2}{\pi}x - 1$

$y = \frac{2}{\pi}\left(\frac{\pi}{8}\right) - 1$

$y = \frac{2}{8} - 1 = -0.75$

actual $\rightarrow f\left(\frac{\pi}{8}\right) = -\frac{\sqrt{3}}{2} = -0.866$

estimate $\rightarrow f\left(\frac{\pi}{8}\right) \approx -0.75$

The estimate of -0.75 is above the actual value -0.866 by 0.116.