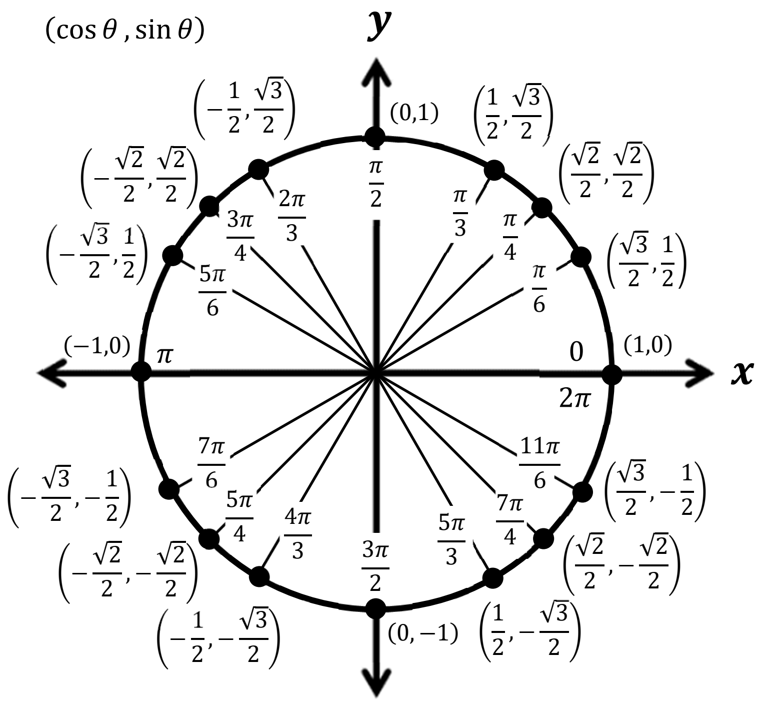
3.4 Sine and Cosine Function Graphs

The Sine Function Graph

Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The sine function, $f(\theta) = \sin\theta$, gives the y-coordinate, or **vertical displacement** from the ***x*-axis,** of point P.



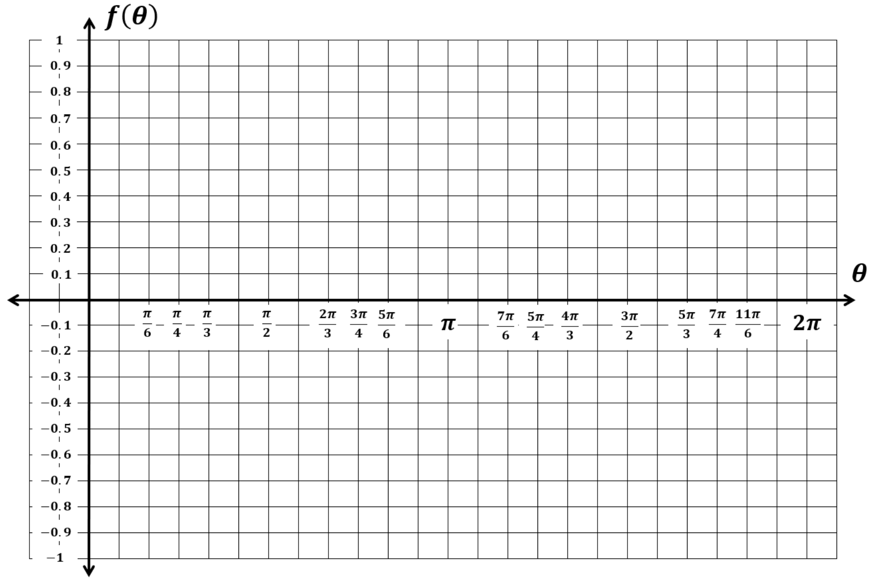
Fill in the table of values for $f(\theta) = \sin\theta$. To save you time, the decimal values have been given.

$\boldsymbol{\theta}$	$\mathbf{0}$	$\boldsymbol{\frac{\pi}{6}}$	$\boldsymbol{\frac{\pi}{4}}$	$\boldsymbol{\frac{\pi}{3}}$	$\boldsymbol{\frac{\pi}{2}}$	$\boldsymbol{\frac{2\pi}{3}}$	$\boldsymbol{\frac{3\pi}{4}}$	$\boldsymbol{\frac{5\pi}{6}}$	$\boldsymbol{\pi}$
$\sin\theta$ (exact)									
$\sin\theta$ (decimal)	0	0.5	0.707	0.866	1	0.866	0.707	0.5	0

$\boldsymbol{\theta}$	$\boldsymbol{\frac{7\pi}{6}}$	$\boldsymbol{\frac{5\pi}{4}}$	$\boldsymbol{\frac{4\pi}{3}}$	$\boldsymbol{\frac{3\pi}{2}}$	$\boldsymbol{\frac{5\pi}{3}}$	$\boldsymbol{\frac{7\pi}{4}}$	$\boldsymbol{\frac{11\pi}{6}}$	$\boldsymbol{2\pi}$
$\sin\theta$ (exact)								
$\sin\theta$ (decimal)	-0.5	-0.707	-0.866	-1	-0.866	-0.707	-0.5	0

Write your questions and thoughts here!

Plot these sine value points and sketch the graph on the axis below.

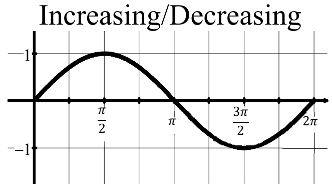


This is considered a periodic function, and you have just graphed one cycle, or period, of the sine function. As we travel around the unit circle the y-values will continue to follow a pattern of increasing to positive one, decreasing to negative one, and then returning to the starting position again of zero.

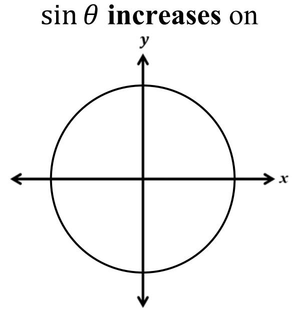
In our next lesson, we will go in depth about how to graph the sine and cosine functions. For this lesson, we will focus on recognizing some basic characteristics of both sine and cosine on the interval $\mathbf{0 \leq \theta \leq 2\pi}$.

Characteristics of the graph of $f(\theta) = \sin\theta$ on the interval $0 < \theta < 2\pi$.

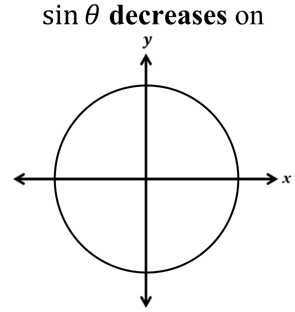
Increasing/Decreasing



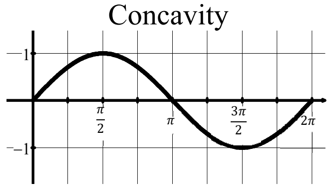
$\sin\theta$ **increases** on



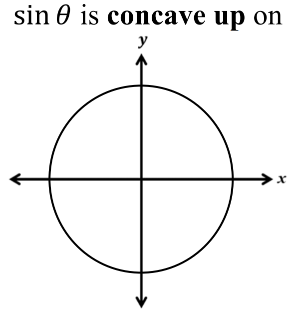
$\sin\theta$ **decreases** on



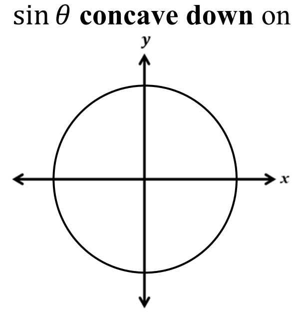
Concavity



$\sin\theta$ is **concave up** on



$\sin\theta$ **concave down** on



Write your questions and thoughts here!

The function f is given by $f(\theta) = \sin\theta$. Describe the concavity of f on the interval, and if f is increasing or decreasing on the interval.

1. $\frac{\pi}{2} < \theta < \pi$	2. $\frac{3\pi}{2} < \theta < 2\pi$	3. $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

The Cosine Function Graph

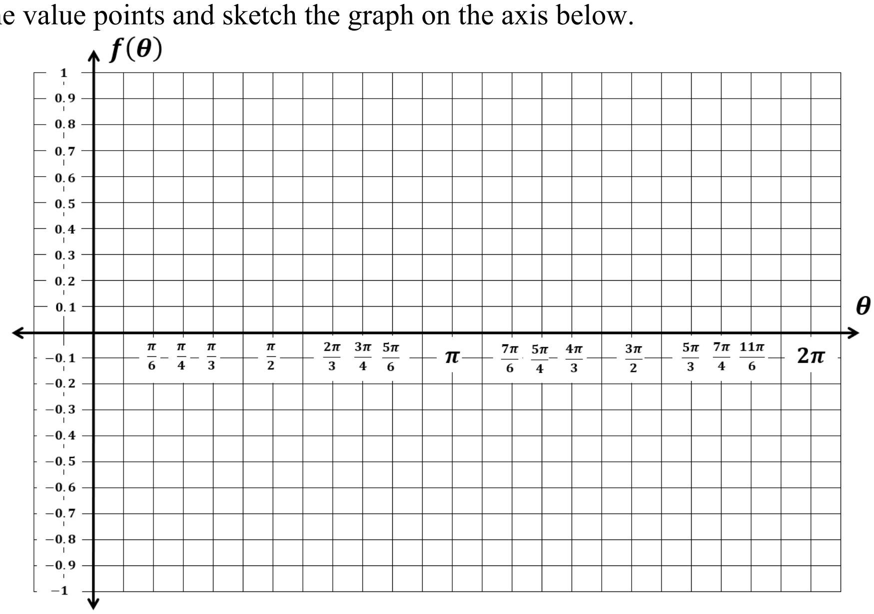
Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine function, $f(\theta) = \cos\theta$, gives the x-coordinate, or **horizontal displacement** from the ***y*-axis,** of point P.

Fill in the table of values for $f(\theta) = \cos\theta$. To save you time, the decimal values have been given.

$\boldsymbol{\theta}$	$\mathbf{0}$	$\boldsymbol{\frac{\pi}{6}}$	$\boldsymbol{\frac{\pi}{4}}$	$\boldsymbol{\frac{\pi}{3}}$	$\boldsymbol{\frac{\pi}{2}}$	$\boldsymbol{\frac{2\pi}{3}}$	$\boldsymbol{\frac{3\pi}{4}}$	$\boldsymbol{\frac{5\pi}{6}}$	$\boldsymbol{\pi}$
$\cos\boldsymbol{\theta}$ **(exact)**									
$\cos\boldsymbol{\theta}$ **(decimal)**	1	0.866	0.707	0.5	0	-0.5	-0.707	-0.866	-1

$\boldsymbol{\theta}$	$\boldsymbol{\frac{7\pi}{6}}$	$\boldsymbol{\frac{5\pi}{4}}$	$\boldsymbol{\frac{4\pi}{3}}$	$\boldsymbol{\frac{3\pi}{2}}$	$\boldsymbol{\frac{5\pi}{3}}$	$\boldsymbol{\frac{7\pi}{4}}$	$\boldsymbol{\frac{11\pi}{6}}$	$\boldsymbol{2\pi}$
$\cos\boldsymbol{\theta}$ **(exact)**								
$\cos\boldsymbol{\theta}$ **(decimal)**	-0.866	-0.707	-0.5	0	0.5	0.707	0.866	1

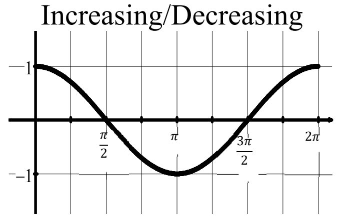
Plot these cosine value points and sketch the graph on the axis below.



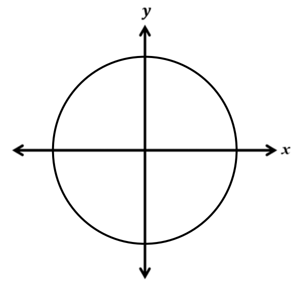
Write your questions and thoughts here!

Characteristics of the graph of $f(\theta) = \cos\theta$ on the interval $0 < \theta < 2\pi$.

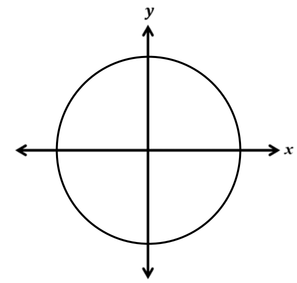
Increasing/Decreasing



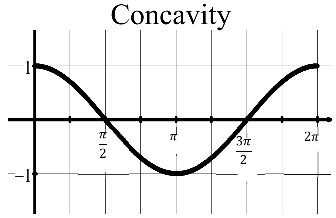
$\cos\theta$ **increases** on



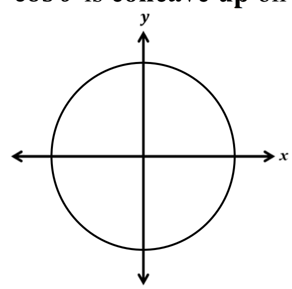
$\cos\theta$ **decreases** on



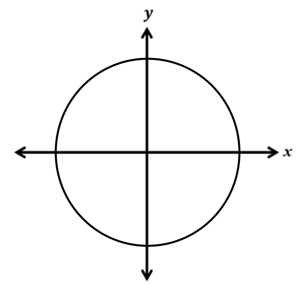
Concavity



$\cos\theta$ is **concave up** on



$\cos\theta$ **concave down** on



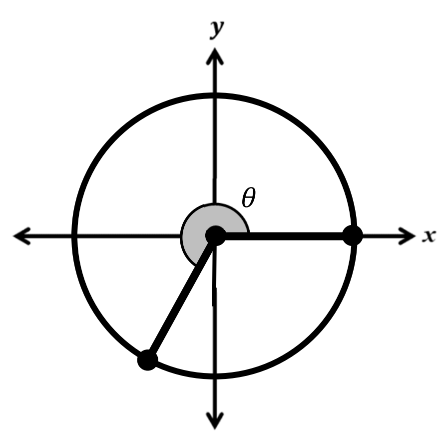
The function f is given by $f(\theta) = \cos\theta$. Describe the concavity of f on the interval, and if f is increasing or decreasing on the interval.

4. $\frac{\pi}{2} < \theta < \pi$	5. $\frac{3\pi}{2} < \theta < 2\pi$	6. $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

7. We are given an angle θ, in standard position as shown in the figure. The function g is given by $g(\alpha) = \sin\alpha$. For the angle α (not shown), $\theta < \alpha < \frac{3\pi}{2}$. Which of the following is true?

(A) $g(\alpha) < g(\theta)$ (B) $g(\alpha) > g(\theta)$ (C) $g(\alpha) = g(\theta)$

(D) Depending on the value of α, sometimes $g(\alpha) < g(\theta)$ and sometimes $g(\alpha) > g(\theta)$.

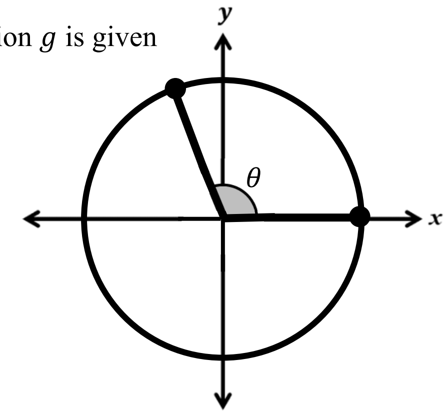


3.4 Sine and Cosine Function Graphs

AP Precalculus

3.4 Practice

1. We are given an angle θ, in standard position as shown in the figure. The function g is given by $g(a) = \cos a$. For the angle α (not shown), $\theta < \alpha < \frac{3\pi}{2}$.
Which of the following is true?



(A) $g(\alpha) < g(\theta)$ (B) $g(\alpha) > g(\theta)$ (C) $g(\alpha) = g(\theta)$

(D) Depending on the value of α, sometimes $g(\alpha) < g(\theta)$ and sometimes $g(\alpha) > g(\theta)$.

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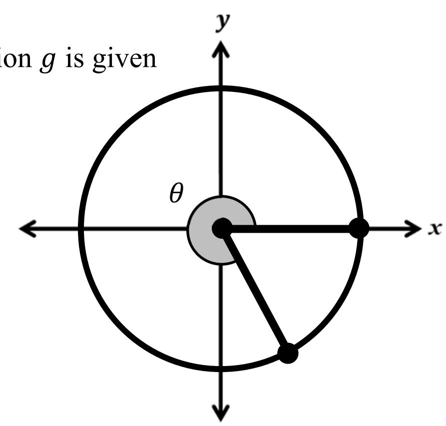
2. We are given an angle θ, in standard position as shown in the figure. The function g is given by $g(a) = \sin a$. For the angle α (not shown), $\theta < \alpha < 2\pi$.
Which of the following is true?

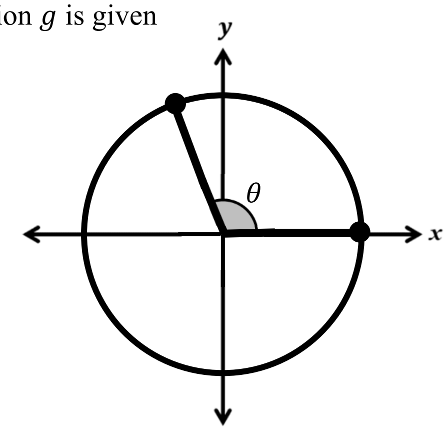


(A) $g(\alpha) < g(\theta)$ (B) $g(\alpha) > g(\theta)$ (C) $g(\alpha) = g(\theta)$

(D) Depending on the value of α, sometimes $g(\alpha) < g(\theta)$ and sometimes $g(\alpha) > g(\theta)$.

---

3. We are given an angle θ, in standard position as shown in the figure. The function g is given by $g(a) = \cos a$. For the angle α (not shown), $\theta < \alpha < 2\pi$.
Which of the following is true?



(A) $g(\alpha) < g(\theta)$ (B) $g(\alpha) > g(\theta)$ (C) $g(\alpha) = g(\theta)$

(D) Depending on the value of α, sometimes $g(\alpha) < g(\theta)$ and sometimes $g(\alpha) > g(\theta)$.

The function f is given by $f(\theta) = \cos\theta$. Describe the concavity of f on the interval, and if f is increasing or decreasing on the interval.

4. $0 < \theta < \frac{\pi}{2}$

5. $\frac{\pi}{2} < \theta < \pi$

6. $\pi < \theta < \frac{3\pi}{2}$

7. $\frac{3\pi}{2} < \theta < 2\pi$

8. $0 < \theta < \pi$

The function f is given by $f(\theta) = \sin\theta$. Describe the concavity of f on the interval, and if f is increasing or decreasing on the interval.

9. $0 < \theta < \frac{\pi}{2}$

10. $\frac{\pi}{2} < \theta < \pi$

11. $\pi < \theta < \frac{3\pi}{2}$

12. $\frac{3\pi}{2} < \theta < 2\pi$

13. $\pi < \theta < 2\pi$

3.4 Sine and Cosine Function Graphs

3.4 Test Prep

14. For the function $f(\theta) = \cos\theta$, what are all values of the domain when $f(\theta) = 1$?

15. For the function $g(\theta) = \sin\theta$, what are all values of the domain when $g(\theta) = 0$?