3.7 Sinusoidal Function Context and Data Modeling

AP Precalculus

Solutions

3.7 Practice

1. The following data set can be modeled by a sinusoidal function. Use the data to answer each problem below.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	97	24	−62	18	104	21	−65	16	101	23

x	10	11	12	13	14	15	16	17	18	19
$f(x)$	−59	22	98	17	−60	19	99	18	−61	23

a. Estimate the period and frequency.

Period = 4

Freq = $\frac{1}{4}$

b. Estimate the vertical shift (midline).

min ≈ −60 max ≈ 100

$\frac{-60+100}{2} = \frac{40}{2}$

up 20

c. Estimate the amplitude.

$\frac{100 - -60}{2} = \frac{160}{2}$

80

d. Using the above information, create a sinusoidal function model.

$\frac{2\pi}{b} = 4$

$\frac{2\pi}{4} = b$ $b = \frac{\pi}{2}$

$f(x) = 80\sin\left(\frac{\pi}{2}[x-3]\right) + 20$ or $f(x) = 80\cos\left(\frac{\pi}{2}x\right) + 20$

Answers may vary. Only one answer needed.

e. Using a calculator, find a sinusoidal model from the given data set. Your answer should look similar to your estimate in part d.

$f(x) = 80.633\sin(1.575x + 1.519) + 19.466$

2. The following data set can be modeled by a sinusoidal function. Use the data to answer each problem below.

x	0	π	2π	3π	4π	5π	6π	7π	8π	9π
$f(x)$	349	220	180	230	345	470	510	465	348	235

x	10π	11π	12π	13π	14π	15π	16π	17π	18π	19π
$f(x)$	207	240	361	458	498	462	358	210	196	215

a. Estimate the period and frequency.

Period = 8π

Freq = $\frac{1}{8\pi}$

b. Estimate the vertical shift (midline).

min ≈ 200 max ≈ 500

$\frac{500+200}{2} = \frac{700}{2}$

up 350

c. Estimate the amplitude.

$\frac{500-200}{2} = \frac{300}{2}$

150

d. Using the above information, create a sinusoidal function model.

$$f(x) = 150\sin\left(\frac{1}{4}[x - 4\pi]\right) + 350$$
$$f(x) = 150\cos\left(\frac{1}{4}[x - 6\pi]\right) + 350$$
$$f(x) = 150\cos\left(\frac{1}{4}[x + 2\pi]\right) + 350$$

Answers may vary. Only one answer needed.

e. Using a calculator, find a sinusoidal model from the given data set. Your answer should look similar to your estimate in part d.

$$f(x) = 161.992\sin(0.25x - 3.12) + 347.497$$

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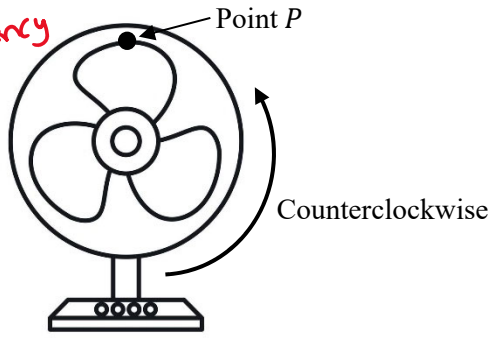
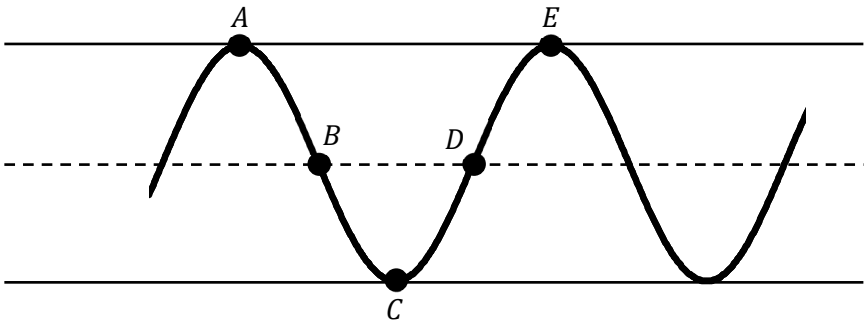
3.7 Test Prep

3. **Calculator active.** Mr. Sullivan is tired of not having air conditioning in Germany, so he buys a large fan for his living room. The blades of this fan rotate in a counterclockwise direction and complete 20 rotations every second. Point P is on the tip of one of the fan blades and is located directly above the center of the fan at time $t = 0$ seconds, as indicated in the figure. Point P is 12 inches from the center of the fan. The center of the fan is 30 inches above the floor. As the fan blades rotate at a constant speed, the distance between P and the floor periodically decreases and increases.

Frequency



The sinusoidal function h models the distance between P and the floor, in inches, as a function of time t, in seconds. The graph of h and its dashed midline for two full cycles is shown. Five points, A, B, C, D, and E, are labeled on the graph. No scale is indicated, and no axes are presented.



Freq = 20

Period = $\frac{1}{20}$ = 0.05

min = 30 − 12 = 18

max = 30 + 12 = 42

amp = 12

Determine possible coordinates $(t, h(t))$ for the five points A, B, C, D, and E

A has coordinates $(0, 42)$
B has coordinates $(0.0125, 30)$
C has coordinates $(0.025, 18)$
D has coordinates $(0.0375, 30)$
E has coordinates $(0.05, 42)$

t coordinates will vary, but one full cycle should be 0.05 seconds

4. A student rides his bicycle to school each day. The pedals are mounted on a bracket whose center is 29 cm above the ground. Each pedal is 16.5 cm from the center of the bracket. Assume that the bicycle is pedaled at 12 cycles per minute. With the starting position of the pedals in a horizontal position at $t = 0$.

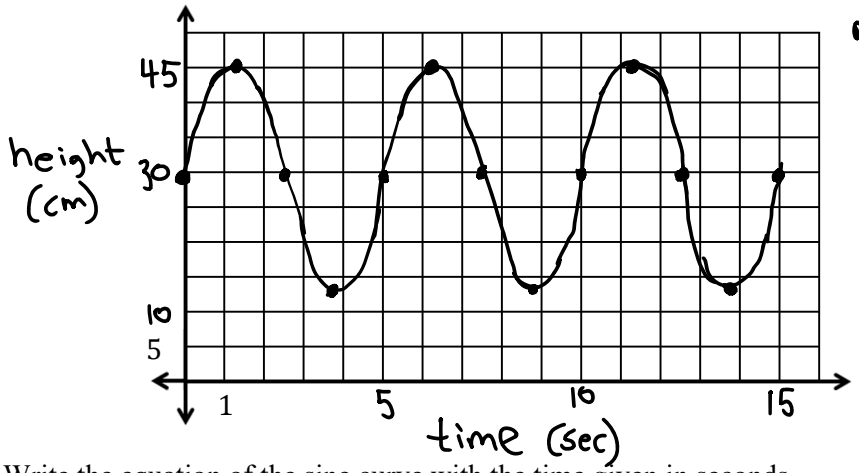
Frequency

Start at $(0, 29)$

a. Label the graph with time (seconds) on the x-axis. Each line will represent one second.

b. Label the y-axis as height (cm) from the ground. Each line will represent 5 cm.

c. Sketch the graph of this sinusoidal function for the first three cycles. Assume the pedal you are measuring goes "up" on the first rotation.



Period = $\frac{1}{12}$ minute or 5 seconds

max = 45.5

min = 12.5

$\frac{2\pi}{b} = 5$

$2\pi = 5b$

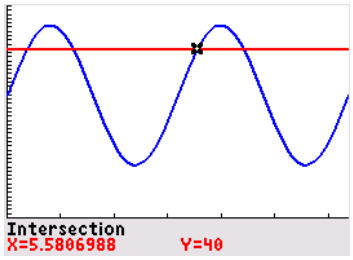
$\frac{2\pi}{5} = b$

d. Write the equation of the sine curve with the time given in seconds.

$y = 16.5 \sin\left(\frac{2\pi}{5}x\right) + 29$

e. **Calculator active.** When is the pedal 40 cm above the ground for the 3rd time?

$t = 5.5806$ seconds



f. **Calculator active.** How high is the pedal after 23 seconds?

$y(23) =$ 19.3015 cm

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5. A robotic ant is designed to cross a table in a sinuisoidal pattern, making a wave pattern as it travels from left to right. The table has a length of 1,800 mm and a width of 800 mm. Standing on one side of the table, the values $x = 0$ and $x = 1{,}800$ represent the left and right sides of the table, respectively. The values $y = 0$ and $y = 800$ represent the nearest and furthest sides of the table, re spectively.

The path of the robotic ant begins on the left side of the table, $x = 0$, and completes one period of a sinusoidal function by ending on the right side of the table, $x = 1{,}800$. During its path, the robotic ant reaches its maximum distance from the near side of the table of $y = 750$ before reaching its minimum distance of $y = 150$. If $y = f(x)$ models the path of the robotic ant, which of the following could define $f(x)$?

$\frac{2\pi}{b} = 1800$

$b = \frac{\pi}{900}$

A

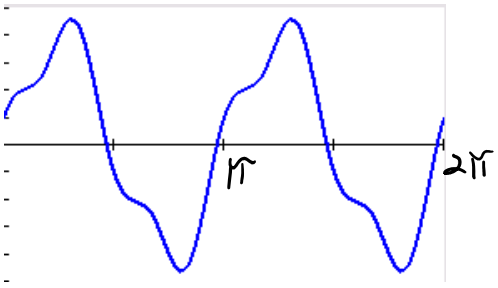
(A) $300 \sin\left(\frac{\pi}{900}x\right) + 450$

(B) $300 \sin(3600\pi x) + 450$

(C) $600 \sin\left(\frac{\pi}{900}x\right) + 450$

(D) $600 \sin(3600\pi x) + 450$

6. **Calculator active.** The function f is given by $f(x) = 4\sin(2x) + \cos(6x)$. Using the period of f, how many number of complete cycles of the graph of f in the xy-plane are on the interval $0 \le x \le 700$? Round to the nearest whole number.



Period is the larger of the two. In this case π. Double-check on the calculator.

$\frac{700}{\pi} \approx 223$ cycles

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7. When an earthquake hits and creates a tsunami, the water first goes down from its normal level, and then rises an equal distance above its normal level, then returns to its normal level. A tsunami is approaching Trig Island and is modeled by:

$$d(t) = 10\sin\left[\frac{2\pi}{15}(t - 7.5)\right] + 11$$

where d is the water's depth in meters and t is the time in minutes since the earthquake. This model only works for one cycle!

a. Using the equation, what is the maximum depth of the tsunami wave? (No calculator!)

21 meters

b. Using the equation, what is the normal depth of water at Trig Island? (No calculator!)

11 meters

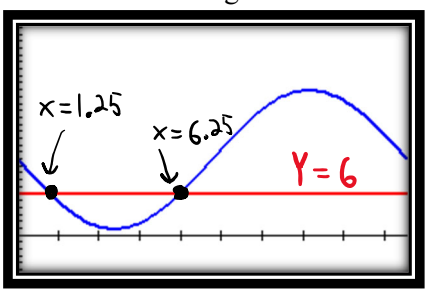
c. What is the period of this function? What does this mean in the context of this problem?

$2\pi \cdot \frac{15}{2\pi} = 15$

It will take 15 minutes for water to return to its normal depth.

d. The Treasure of Trig Island lies 5 meters beneath the normal surface of the water. For how long will the Treasure of Trig Island be exposed? (Remember to look at your answer from part b.)

$6.25 - 1.25 =$ 5 minutes



8. The function g is given by $f(\theta) = \sin(\theta)$. The sinusoidal function h is a phase shift of the function g by positive $\frac{\pi}{6}$ units. Which of the following is true?

sin has minima at $\theta = -\frac{\pi}{2}$ and $\frac{3\pi}{2}$

$-\frac{\pi}{2} + \frac{\pi}{6}$

$-\frac{3\pi}{6} + \frac{\pi}{6} = -\frac{2\pi}{6} = -\frac{\pi}{3}$

$\frac{3\pi}{2} + \frac{\pi}{6}$

$\frac{9\pi}{6} + \frac{\pi}{6} = \frac{10\pi}{6} = \frac{5\pi}{3}$

(A) Consecutive minima of h occur at $\left(-\frac{5\pi}{6}, g\left(-\frac{5\pi}{6}\right)\right)$ and $\left(\frac{7\pi}{6}, g\left(\frac{7\pi}{6}\right)\right)$ because consecutive minima of g occur at $(-\pi, g(-\pi))$ and $(\pi, g(\pi))$, and h is the image of g with a horizontal shift of $\frac{\pi}{6}$ units right.

(B) Consecutive minima of h occur at $\left(-\frac{7\pi}{6}, g\left(-\frac{7\pi}{6}\right)\right)$ and $\left(\frac{5\pi}{6}, g\left(\frac{5\pi}{6}\right)\right)$ because consecutive minima of g occur at $(-\pi, g(-\pi))$ and $(\pi, g(\pi))$, and h is the image of g with a horizontal shift of $\frac{\pi}{6}$ units left.

C

(C) Consecutive minima of h occur at $\left(-\frac{\pi}{3}, g\left(-\frac{\pi}{3}\right)\right)$ and $\left(\frac{5\pi}{3}, g\left(\frac{5\pi}{3}\right)\right)$ because consecutive minima of g occur at $\left(-\frac{\pi}{2}, g\left(-\frac{\pi}{2}\right)\right)$ and $\left(\frac{3\pi}{2}, g\left(\frac{3\pi}{2}\right)\right)$, and h is the image of g with a horizontal shift of $\frac{\pi}{6}$ units right.

(D) Consecutive minima of h occur at $\left(-\frac{2\pi}{3}, g\left(-\frac{2\pi}{3}\right)\right)$ and $\left(\frac{4\pi}{3}, g\left(\frac{4\pi}{3}\right)\right)$ because consecutive minima of g occur at $\left(-\frac{\pi}{2}, g\left(-\frac{\pi}{2}\right)\right)$ and $\left(\frac{3\pi}{2}, g\left(\frac{3\pi}{2}\right)\right)$, and h is the image of g with a horizontal shift of $\frac{\pi}{6}$ units left.

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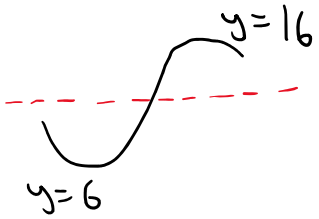
9. The function f is defined by $f(x) = a\sin(b(x + c) + d$. In the xy-plane, the points $(3,6)$ and $(5,16)$ represent a minimum value and a maximum value, respectively, on the graph of f. What are values of a and d?



midline $d = 11$

amplitude $a = 5$