

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Unit 2A CA – Exponential and Logarithmic Functions**

Find an equation that gives the  $n$ th term of each sequence. Instead of the initial value use the  $k$ th term of the sequence in your equation.  $k$  is given for each problem.

1.  $\{-2, -8, -32, -128, \dots\}$   $k = 1$

2.  $\{325, 225, 125, 25, \dots\}$   $k = 4$

3.  $\{-25, -19, -13, -7, \dots\}$   $k = 0$

4.  $\{16, 4, 1, \frac{1}{4}, \dots\}$   $k = 3$

Find the  $n$ th term of each sequence. Write an equation for each sequence before finding the  $n$ th term.

5.  $\{117, 106, 95, 84, \dots\}$  What is the 47<sup>th</sup> term?

6.  $\{-24, -19, -14, -9, \dots\}$  What is the 401<sup>st</sup> term?

A function has the following coordinate points. Could the function represent a linear function, exponential function, or neither?

7.  $(4, 6), (5, 10), (6, 18)$

8.  $(1, 3), (2, 12), (3, 48)$

9.  $(16, -5), (17, -1), (18, 3)$

The following functions are either linear or exponential. Which is it? Justify your answer.

10.

$x$	2	7	12
$f(x)$	3	12	48

11.

$x$	2	6	10
$f(x)$	12	5	-2

Is each function linear or exponential. Identify the constant (slope or ratio) that causes the output values to change?

12.  $y = \left(\frac{2}{3}\right)^x$

13.  $y = 9x - 1$

14.  $y - 6 = \frac{1}{4} \cdot 2^{x-5}$

15.  $y + 5 = 3(x + 6)$

It is known that  $f(x)$  is a linear function and that it passes through the given points. Write an equation for this function.

16. (8, 85) and (13, 5)

17. (2, 8) and (4, 20)

It is known that  $f(x)$  is an exponential function and that it passes through the given points. Write an equation for this function.

18. (8, 85) and (13, 5)

19. (2, 8) and (4, 20)

20. Identify if the exponential function  $f(x) = 0.6(5.25)^x$  is growth or decay and justify your response.

21. The value  $3 \cdot 3 \cdot 3 \cdot 3 \cdot 7$  is an output value of an exponential function of the form  $f(x) = a \cdot b^x$ , where  $a$  and  $b$  are constants. Write the function along with the input value that represents the output value.

$f(x) =$

where  $x =$

<p>22. Is the exponential function <math>f(x) = -4(0.6)^x</math> increasing or decreasing?</p>	<p>23. Is the exponential function <math>f(x) = 105\left(\frac{2}{3}\right)^x</math> concave up or concave down?</p>	<p>24. If <math>f(x) = -\frac{4}{5}(7)^x</math>, find...</p> <p>a. <math>\lim_{x \rightarrow -\infty} f(x) =</math></p> <p>b. <math>\lim_{x \rightarrow \infty} f(x) =</math></p>
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**Let  $f(x)$  be a function on which a transformation occurs. Let  $g(x)$  be a transformation of  $f$ . For each problem, name the transformation(s) of  $f$ .**

<p>25. <math>f(x) = 2^x</math> and <math>g(x) = f(x) \cdot 16</math></p>	<p>26. <math>f(x) = 4^x</math> and <math>g(x) = (f(x))^{-2}</math></p>
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<p>27. <math>f(x) = 2^x</math> and <math>g(x) = \frac{f(x)}{4}</math>.</p>	<p>28. <math>f(x) = 2^x</math> and <math>g(x) = -\frac{f(x)}{16}</math></p>
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**Evaluate the function at the given input values.**

<p>29. Let <math>h(x) = 10 \cdot 5^{x/4}</math>. Find <math>h(2)</math></p>	<p>30. Let <math>h(x) = 4 \cdot 8^{x/3}</math>. Find <math>h(-1)</math></p>
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31. Below is a table of values for an exponential function in the form  $f(x) = a(b)^x + k$ . Write the equation for this exponential function.

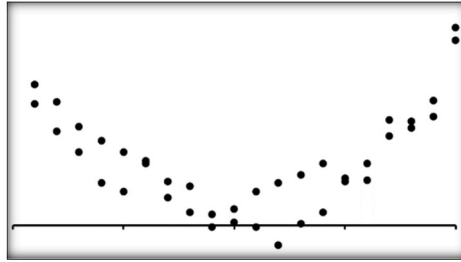
$x$	0	1	2	3	4
$f(x)$	4	11	25	53	109

32. After a long weekend away, Mr. Brust comes home to find he left sliced bananas on the countertop, and fruit flies are all over his kitchen. Using his security cameras, he estimates the number of fruit flies in his kitchen at selected values of  $h$ , hours after Saturday morning at 8:00 a.m.

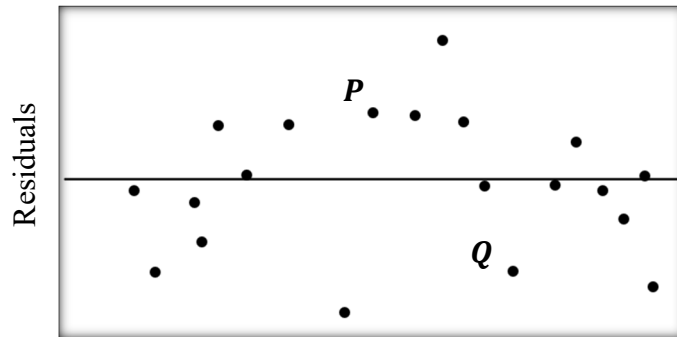
Hours ( $h$ )	1	10	15	24
Population of fruit flies	2	20	50	200

- Use an exponential regression  $F(h) = ab^h$  to model these data. Round to three decimals but store the original equation in your calculator.
- According to the model in your calculator, how many fruit flies were there after 12 hours?
- If the model were to continue, how many fruit flies would be in Mr. Brust's kitchen if he was gone for 3 days? Round to the nearest whole number.
- How long before there are 1,000 fruit flies? Round to the nearest whole number.
- Use your model from part *a* to find the error at  $t = 10$ .
- Is the value predicted an overestimate or underestimate of the actual value?

33. A student creates a model for a data set. The residual plot for their quadratic regression is shown. Is the model appropriate? Why or why not?



34. Mr. Sullivan loves going to the movies, and realizes that the more he makes, the more often he goes to the movie theater. He calculated a regression model with his yearly salary as input values and the amount he spends at a movie theater each year as the output values. The residual plot for the regression model and data points is below.



- a. The given residual plot has a point labeled  $P$  at the coordinate  $(67000, 37.09)$ . What does point  $P$  indicate in the context?
- b. The point labeled  $Q$  is at the coordinate  $(77000, -51.91)$ . What does point  $Q$  indicate in the context?

Let  $f(x) = 4^x$  and  $g(x) = -3 - x$ .

35. Find  $(f \circ g)(-2)$ .

36. Find  $g\left(f\left(\frac{1}{2}\right)\right)$ .

37. Find  $g(f(2))$ .

38. Let  $f(x) = 2^x + 1$  and  $h(x) = f(g(x))$ . Fill in the table.

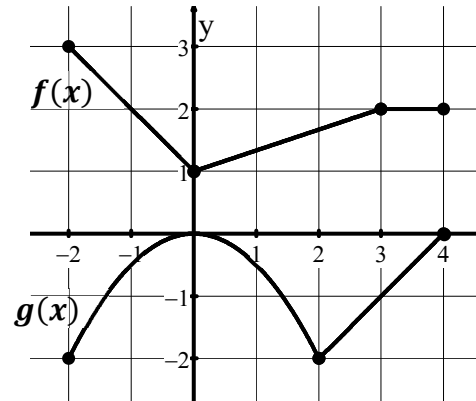
$x$	$g(x)$	$h(x)$
1	-1	
2	0	
3	2	

Use the graphs of  $f$  and  $g$  to find the given values.

39. Find  $f(g(0))$ .

40. Find  $g(f(3))$ .

41. Find  $g(f(-2))$ .



Let  $f(x) = \sqrt{2x - 1}$  and  $g(x) = 1 - x^2$ .

42. Find  $f \circ g$

44. Find  $g \circ f$

43. State the domain of  $f \circ g$

45. State the domain of  $g \circ f$

46. Express  $h(x) = e^{1-3x}$  as a composition of two simpler functions  $f$  and  $g$  where  $h(x) = f(g(x))$ .

$$f(x) =$$

$$g(x) =$$

Find the inverse of each function. List the domain and range of  $f^{-1}(x)$ .

47.  $f(x) = -\sqrt{x-2} + 7$

Domain of  $f^{-1}(x)$ :

Range of  $f^{-1}(x)$ :

48.  $f(x) = \frac{6}{x-1}$

Domain of  $f^{-1}(x)$ :

Range of  $f^{-1}(x)$ :

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49.  $f(x) = (x+9)^2 + 6$  for  $x \geq -9$

Domain of  $f^{-1}(x)$ :

Range of  $f^{-1}(x)$ :

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50. Use the table to find the given values.

a.  $f(6) =$

b.  $f(11) =$

c.  $f^{-1}(8) =$

d.  $f^{-1}(6) =$

e.  $f(9) =$

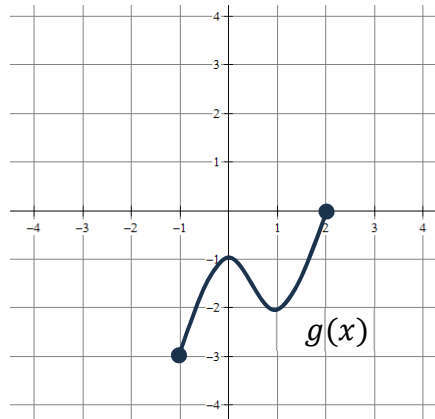
f.  $f^{-1}(9) =$

$x$	$f(x)$
6	8
7	10
8	6
9	11
10	9
11	7

51.

a. Is  $g(x)$  invertible?

b. Sketch the graph of  $g^{-1}(x)$ , regardless of whether or not it is invertible.





## Answers to Unit 2A Corrective Assignment

1. $g_n = -2(4)^{n-1}$	2. $g_n = 25 - 100(n - 4)$	3. $g_n = -31 + 6n$	4. $g_n = \left(\frac{1}{4}\right)^{n-3}$	5. $-389$					
6. 1976	7. neither	8. exponential	9. linear	10. Exponential because for each input change of 5, $f$ changes proportionally by a ratio of 4.					
11. Linear because for each input change of 4, $f$ changes at a constant rate $-7$ .		12. Ratio of $\frac{2}{3}$	13. Slope of 9	14. Ratio of 2	15. Slope of 3				
16. $y - 85 = -16(x - 8)$ or $y - 5 = -16(x - 13)$		17. $y - 8 = 6(x - 2)$ or $y - 20 = 6(x - 4)$		18. $y = 85 \cdot \left(\sqrt[5]{\frac{1}{17}}\right)^{x-8}$ or $y = 5 \cdot \left(\sqrt[5]{\frac{1}{17}}\right)^{x-13}$		19. $y = 8 \cdot \left(\sqrt{\frac{5}{2}}\right)^{x-2}$ or $y = 20 \cdot \left(\sqrt{\frac{5}{2}}\right)^{x-4}$			
20. Growth because $b > 1$		21. $f(x) = 7(3)^x$ where $x = 4$		22. Increasing		23. Concave up			
24. a. $\lim_{x \rightarrow -\infty} f(x) = 0$ b. $\lim_{x \rightarrow \infty} f(x) = -\infty$		25. Shift left 4 units.		26. Horizontal dilation and reflection across the $y$ -axis.		27. Shift right 2 units.			
28. Reflection across the $x$ -axis and shift right 4.		29. $10^4\sqrt[25]{5}$		30. $\frac{4}{\sqrt[3]{8}}$		31. $f(x) = 7(2)^x - 3$			
32a. $F(h) = 2.078(1.221)^x$ 32b. 22.762 32c. 3,586,274 32d. $x = 30.965$ . 31 hours 32e. $15.304 - 20 = -4.696$ 32f. Underestimate		33. No. The residual plot creates a pattern.		34a. Because point $P$ is above the $x$ -axis, for the year Mr. Sullivan's salary was \$67,000, the model underestimates the money spent at a movie theater by \$37.09. 34b. Because point $Q$ is below the $x$ -axis, for the year Mr. Sullivan's salary was \$77,000, the model overestimates the money spent at a movie theater by \$51.91.					
35. $\frac{1}{4}$	36. $-5$	37. $-19$	38a. $\frac{3}{2}$ 38b. 2 38c. 5	39. 1	40. $-2$	41. $-1$	42. $\sqrt{1 - 2x^2}$		
43. $\left[-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\right]$		44. $2 - 2x$		45. $\left[\frac{1}{2}, \infty\right)$		46. $f(x) = e^x$ $g(x) = 1 - 3x$		47. $f^{-1}(x) = (x - 7)^2 + 2$ Domain: $x \leq 7$ Range: $y \geq 2$	
48. $f^{-1}(x) = \frac{6}{x} + 1$ Domain: $\mathbb{R}, x \neq 0$ Range: $\mathbb{R}, y \neq 1$		49. $f^{-1}(x) = \sqrt{x - 6} - 9$ Domain: $x \geq 6$ Range: $y \geq -9$		50a. 8 50b. 7 50c. 6 50d. 8 50e. 11 50f. 10		51. Not invertible. 