

Name: Solutions

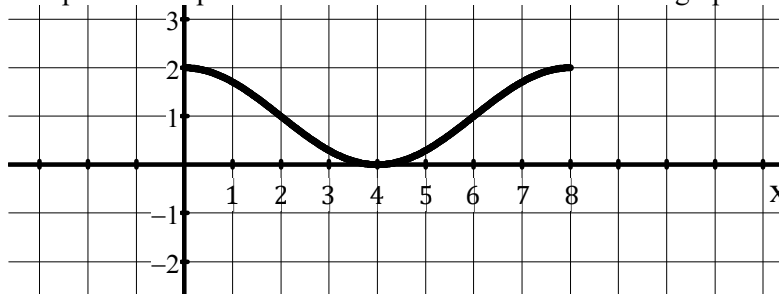
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**Unit 3A Review – Trigonometric and Polar Functions**

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets for lessons 3.1 – 3.7.

1. The graph below shows one period of a periodic function. Sketch the rest of the graph on the given axes.



a. Is the function above increasing, decreasing or both on the interval  $20 < x < 24$ .

$20 < x < 24$  is the same as  $4 < x < 8$  on this periodic function. Therefore, it is **INCREASING**.

**The measurement of an angle in standard position is listed. In which quadrant is the terminal ray?**

2.  $6.6\pi$

$$6.6\pi - 6\pi = 0.6\pi$$

Quadrant II

3.  $-2.3\pi$

$$-2.3\pi + 4\pi = 1.7\pi$$

Quadrant IV

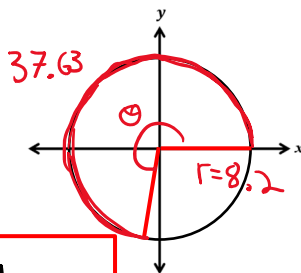
**Below are various measurements of a circle's radius, an angle within the circle, or the arc subtended by the angle. SKETCH the approximate angle on the axes and find the missing value.**

4. Radius is 8.2 and the length of an arc subtended by an angle is 37.63. Find the measure of the angle.

$$\theta = \frac{s}{r}$$

$$\theta = \frac{37.63}{8.2}$$

$$\theta = 4.589 \text{ radians}$$

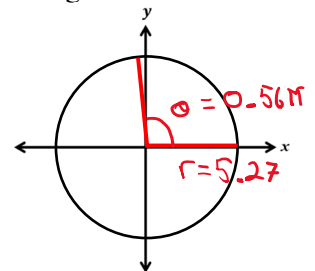


5. Radius is 5.27 and an angle is  $0.56\pi$  radians. Find the length of the arc subtended by the angle.

$$\theta = \frac{s}{r}$$

$$0.56\pi = \frac{s}{5.27}$$

$$s = 9.271$$



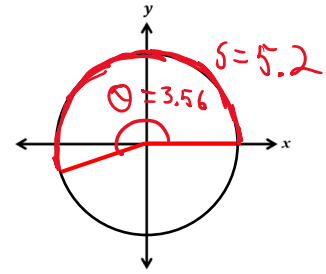
6. An angle is 3.56 radians and the length of an arc subtended by the angle is 5.2. What is the radius of the circle?

$$\theta = \frac{s}{r}$$

$$3.56 = \frac{5.2}{r}$$

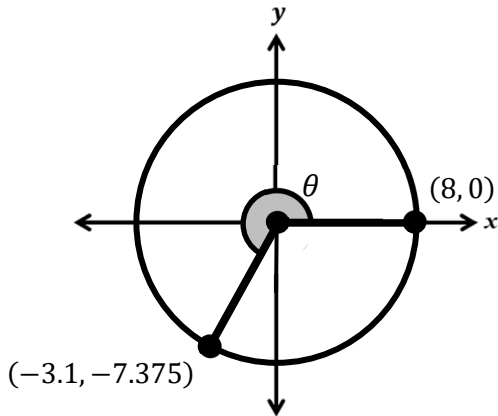
$$r = \frac{5.2}{3.56}$$

$$r = 1.4606$$



Each figure below gives a circle in the  $xy$ -plane with center at the origin, and an angle  $\theta$  in standard position. Find the value of each expression.

7.

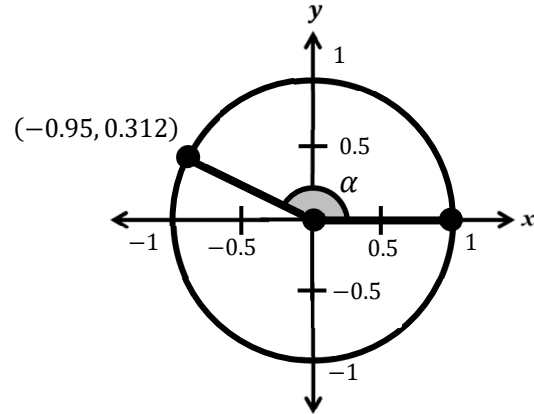


a.  $\sin \theta = -\frac{7.375}{8} \approx -0.9218$

b.  $\cos \theta = -\frac{3.1}{8} = -0.3875$

c.  $\tan \theta = \frac{-7.375}{-3.1} \approx 2.379$

8.



a.  $\sin \alpha = 0.312$

b.  $\cos \alpha = -0.95$

c.  $\tan \alpha = \frac{0.312}{-0.95} \approx -0.328$

Find the value of each expression. Do not look back at the Unit Circle for help!

9.  $\cos \pi$

$$-1$$

10.  $\sin \frac{\pi}{4}$

$$\frac{\sqrt{2}}{2}$$

11.  $\cos \frac{2\pi}{3}$

$$-\frac{1}{2}$$

12.  $\cos \frac{7\pi}{6}$

$$-\frac{\sqrt{3}}{2}$$

13.  $\sin \left(-\frac{\pi}{6}\right)$

$$-\frac{1}{2}$$

14.  $\cos \left(-\frac{5\pi}{4}\right)$

$$-\frac{\sqrt{2}}{2}$$

For each problem, an angle in standard position in the  $xy$ -plane is given in radians. A circle is centered at the origin with the given radius. What are the coordinates of the point of intersection of the terminal ray of the angle and the circle?

15.  $\theta = \frac{3\pi}{2}, r = 6$   $(r \cos \theta, r \sin \theta)$

$x = 6 \cdot \cos \frac{3\pi}{2} = 6 \cdot (0) = 0$   
 $y = 6 \cdot \sin \frac{3\pi}{2} = 6 \cdot (-1) = -6$

$(0, -6)$

16.  $\theta = \frac{5\pi}{3}, r = 10$

$x = 10 \cdot \cos(\frac{5\pi}{3}) = 10 \cdot (\frac{1}{2}) = 5$   
 $y = 10 \cdot \sin(\frac{5\pi}{3}) = 10 \cdot (-\frac{\sqrt{3}}{2}) = -5\sqrt{3}$

$(5, -5\sqrt{3})$

In the  $xy$ -plane, the terminal ray of angle  $\theta$  in standard position intersects a circle of radius  $r$  at the given point. What are the values of  $\theta$  and  $r$ , with  $0 \leq \theta \leq 2\pi$ ?

17.  $(\frac{\sqrt{3}}{2}, \frac{-1}{2})$   
 $(2 \cdot \frac{\sqrt{3}}{2}, 2 \cdot (-\frac{1}{2}))$   $\cos \theta = \frac{\sqrt{3}}{2}$   
 $\sin \theta = -\frac{1}{2}$

$\theta = \frac{11\pi}{6}, r = 2$

18.  $(\frac{-6}{12}, \frac{6\sqrt{3}}{12})$   
 $(12 \cdot -\frac{1}{2}, 12 \cdot \frac{\sqrt{3}}{2})$   $\cos \theta = -\frac{1}{2}$   
 $\sin \theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{2\pi}{3}, r = 12$

The function  $f$  is given by  $f(\theta) = \cos \theta$ . Describe the concavity of  $f$  on the interval, and if  $f$  is increasing or decreasing on the interval.



Concave up  
Decreasing

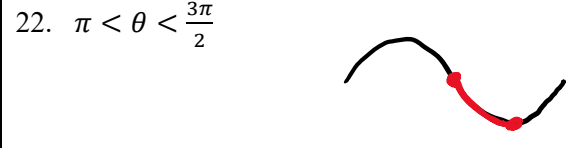


Concave down  
Increasing

The function  $f$  is given by  $f(\theta) = \sin \theta$ . Describe the concavity of  $f$  on the interval, and if  $f$  is increasing or decreasing on the interval.



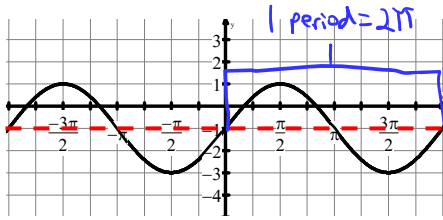
Concave down  
Increasing



Concave up  
Decreasing

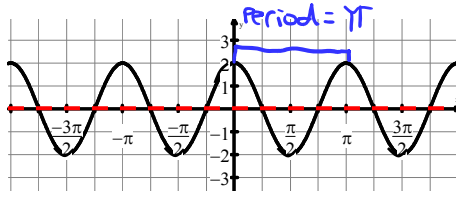
Write the given function for each graph. Use a positive coefficient for all equations.

23. Write a sine function.



$$y = 2 \sin x - 1$$

24. Write a cosine function.

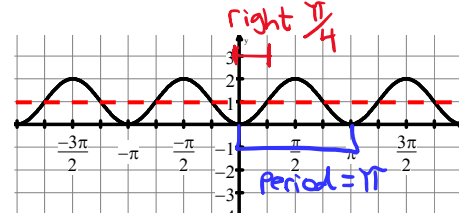


$$\frac{2\pi}{\phi} = \pi$$

$$2 = b$$

$$y = 2 \cos(2x)$$

25. Write a sine function.



$$\frac{2\pi}{\phi} = \pi$$

$$2 = b$$

$$y = \sin(2x - \frac{\pi}{2}) + 1$$

State the amplitude, period, phase shift, and vertical shift.

26.  $y = 3 \cos(5x - \pi)$   
 $5(x - \frac{\pi}{5})$

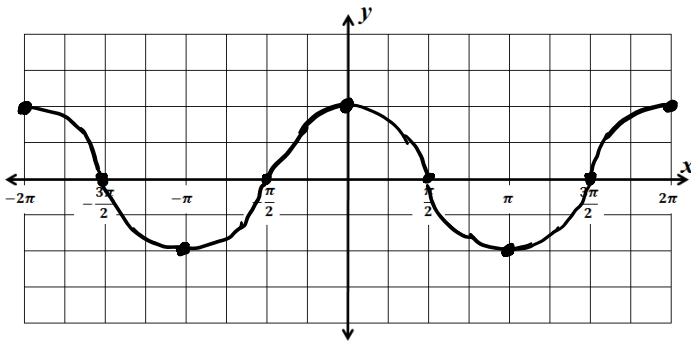
Amp: 3      Period:  $\frac{2\pi}{5}$   
 Midline:  $y=0$       Freq:  $\frac{5}{2\pi}$   
 Max value: 3      Min value: -3  
 Phase shift: right  $\frac{\pi}{5}$

27.  $y = \frac{1}{2} \sin(4x) - 2$

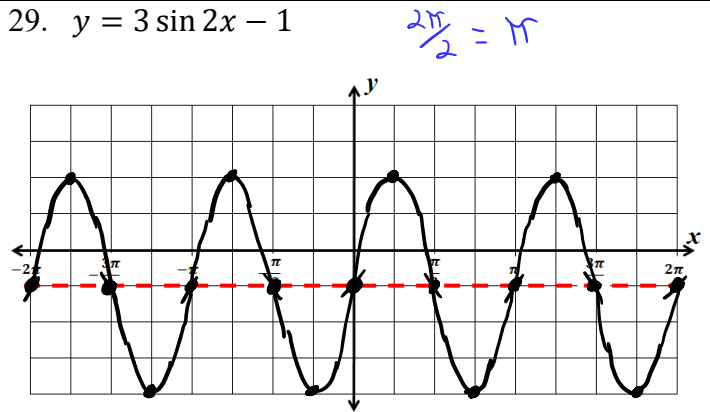
Amp:  $\frac{1}{2}$       Period:  $\frac{2\pi}{4} = \frac{\pi}{2}$   
 Midline:  $y=-2$       Freq:  $\frac{2}{\pi}$   
 Max value: -1.5      Min value: -2.5  
 Phase shift: none

Graph each function on the given set of axes.

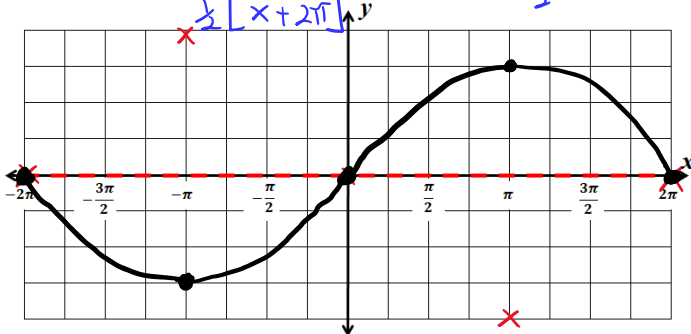
28.  $y = 2 \cos x$



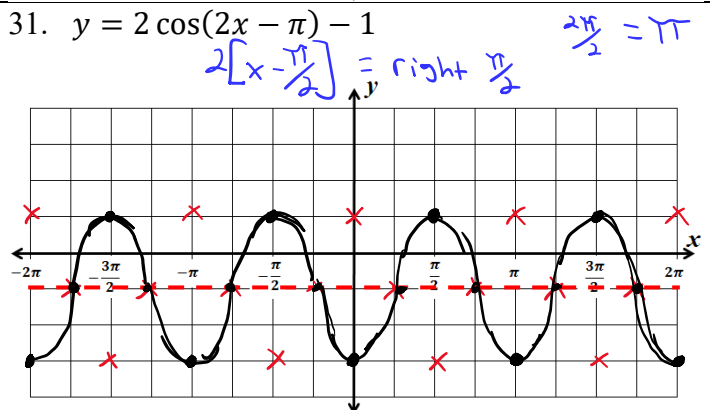
29.  $y = 3 \sin 2x - 1$



30.  $y = -3 \sin(\frac{x}{2} + \pi)$



31.  $y = 2 \cos(2x - \pi) - 1$



32. Suppose you are riding a Ferris wheel. After everyone is loaded, the wheel starts to turn, and the ride lasts for 105 seconds. Your height  $h$  (in feet) above the ground at any time  $t$  (in seconds) can be modeled by the equation  $h(t) = 50 \sin\left[\frac{\pi}{10}(t - 4)\right] + 65$ . You do not need a calculator for any question except the last two.

a. What is the period?  $\frac{2\pi}{\pi/10} = 20$

b. What does the period represent? It takes 20 seconds to make one full rotation.

c. What is the frequency?  $\frac{1}{20}$

d. What does the frequency represent? The ride makes  $\frac{1}{20}$  of a rotation every second.

e. What is your maximum height?  $65 + 50 = 115$  feet

f. What is your minimum height?  $65 - 50 = 15$  feet

g. How many circles will the Ferris Wheel make during the ride?  $\frac{105}{20} = 5.25$  circles

h. **Calculator active.** How high are you when the ride begins?  $h(0) = 17.447$  feet

i. **Calculator active.** What is your height when the ride stops?  $h(105) = 80.451$  feet

33. The following data set can be modeled by a sinusoidal function. Use the data to answer each problem below.

$x$	0	1	2	3	4	5	6	7	8	9
$f(x)$	16	2	-10	-1	18	21	40	52	37	29

min

$x$	10	11	12	13	14	15	16	17	18	19
$f(x)$	12	-2	-11	4	17	22	38	49	39	26

min

a. Estimate the period and frequency.  
 Period: 10  
 Frequency:  $\frac{1}{10}$

b. Estimate the vertical shift (midline).  
 up 20

c. Estimate the amplitude.  
 30

d. Using the above information, create a sinusoidal function model.  
 $f(x) = 30 \sin\left(\frac{\pi}{5}[x - 4.5]\right) + 20$  or  $f(x) = 30 \cos\left(\frac{\pi}{5}[x - 7]\right) + 20$

e. Using a calculator, find a sinusoidal model from the given data set. Your answer should look similar to your estimate in part d.  
 $f(x) = 25.683 \sin(0.636x - 2.903) + 19.899$