10.2 Practice – Phase Shift and Tangent

Pre-Calculus

For 1-3, identify the amplitude, period, phase shift and vertical shift of each function.

1. \( f(x) = 13 \sin \left( 4 \left( x + \frac{\pi}{11} \right) \right) - 7 \)
   - Amp: \( 13 \)  
   - Period: \( \frac{2\pi}{4} = \frac{\pi}{2} \)  
   - Phase Shift: \( \text{left} \frac{\pi}{11} \)  
   - Vertical Shift: \( \text{down} 7 \)

2. \( f(\theta) = -25 \cos(7(\theta - 10)) + 3 \)
   - Amp: \( 25 \)  
   - Period: \( \frac{2\pi}{7} \)  
   - Phase Shift: \( \text{right} 10 \)  
   - Vertical Shift: \( \text{up} 3 \)

3. \( f(\theta) = 4 - 9 \sin \left( 2 \left( \theta + \frac{2\pi}{3} \right) \right) \)
   - Amp: \( 9 \)  
   - Period: \( \frac{2\pi}{2} = \pi \)  
   - Phase Shift: \( \text{left} \frac{2\pi}{3} \)  
   - Vertical Shift: \( \text{up} 4 \)

4. \( f(\theta) = -3 \cos \left( \frac{\theta + \pi}{3} \right) - 10 \)
   - Amp: \( 3 \)  
   - Period: \( \frac{2\pi}{\frac{\theta + \pi}{3}} = \frac{6\pi}{\theta + \pi} \)  
   - Phase Shift: \( \text{left} \frac{\pi}{8} \)  
   - Vertical Shift: \( \text{down} 10 \)

5. \( f(\theta) = 2 \sin \left( \frac{\theta - \pi}{2} \right) - 5 \)
   - Amp: \( 2 \)  
   - Period: \( \frac{2\pi}{\frac{\theta - \pi}{2}} = \frac{4\pi}{\theta - \pi} \)  
   - Phase Shift: \( \text{right} \frac{2\pi}{5} \)  
   - Vertical Shift: \( \text{down} 5 \)

6. \( f(x) = 6 \cos \left( 3(x + \frac{\pi}{4}) \right) + 11 \)
   - Amp: \( 6 \)  
   - Period: \( \frac{2\pi}{3} \)  
   - Phase Shift: \( \text{left} \frac{\pi}{12} \)  
   - Vertical Shift: \( \text{up} 11 \)

For 7-18, graph the trig function.

7. \( y = \sin \left( x + \frac{\pi}{4} \right) \)
   - Amp: \( 1 \)  
   - Period: \( \frac{2\pi}{1} = 2\pi \)  
   - Phase Shift: \( \text{left} \frac{\pi}{4} \)  
   - Vertical Shift: \( \text{none} \)

8. \( y = -\cos \left( x - \frac{\pi}{2} \right) \)
   - Amp: \( 1 \)  
   - Period: \( \frac{2\pi}{1} = 2\pi \)  
   - Phase Shift: \( \text{right} \frac{\pi}{2} \)  
   - Vertical Shift: \( \text{none} \)

9. \( y = 3 \cos \left( x - \frac{3\pi}{4} \right) - 1 \)
   - Amp: \( 3 \)  
   - Period: \( \frac{2\pi}{1} = 2\pi \)  
   - Phase Shift: \( \text{right} \frac{3\pi}{4} \)  
   - Vertical Shift: \( \text{down} 1 \)

10. \( y = 2 + \sin \left( 2 \left( x + \frac{\pi}{4} \right) \right) \)
    - Amp: \( 1 \)  
    - Period: \( \frac{2\pi}{2} = \pi \)  
    - Phase Shift: \( \text{left} \frac{\pi}{4} \)  
    - Vertical Shift: \( \text{up} 2 \)

11. \( y = -\sin \left( \frac{1}{2} \left( x - \frac{\pi}{2} \right) \right) - 2 \)
    - Amp: \( 1 \)  
    - Period: \( \frac{2\pi}{\frac{1}{2}} = 4\pi \)  
    - Phase Shift: \( \text{right} \frac{\pi}{2} \)  
    - Vertical Shift: \( \text{down} 2 \)

12. \( y = 2 \cos \left( 4(x - \pi) \right) - 1 \)
    - Amp: \( 2 \)  
    - Period: \( \frac{2\pi}{4} = \frac{\pi}{2} \)  
    - Phase Shift: \( \text{right} \frac{\pi}{2} \)  
    - Vertical Shift: \( \text{down} 1 \)
13. $y = \frac{1}{2} \sin \left( x - \frac{3\pi}{2} \right)$

Amp: $\frac{1}{2}$
Period: $2\pi$
Phase Shift: right $\frac{3\pi}{2}$
Vertical Shift: none

14. $y = -2 \sin (2x + \pi) + 1$

Amp: $2$
Period: $\pi$
Phase Shift: left $\frac{\pi}{2}$
Vertical Shift: up

15. $y = \cos \left( 2x - \frac{\pi}{2} \right)$

Amp: $1$
Period: $\pi$
Phase Shift: right $\frac{\pi}{4}$
Vertical Shift: none

16. $y = -\frac{1}{2} \tan x$

Period: $\frac{\pi}{2}$
Phase Shift: none
Vertical Shift: none

17. $y = 4 \tan \left( -\frac{1}{2}x \right) - 3$

Period: $\pi$
Phase Shift: none
Vertical Shift: down $3$

18. $y = \tan(2x - \pi) + 2$

Period: $\frac{\pi}{2}$
Phase Shift: right $\frac{\pi}{4}$
Vertical Shift: up

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For 19–21, use the given information to create a sine function.

19.
Amplitude: 2
Period: $\frac{3\pi}{2}$
Phase Shift: left $\frac{5\pi}{6}$
Vertical Shift: down 14

$y = 2 \sin \left( \frac{4}{3}(x + \frac{5\pi}{6}) \right) - 14$

20.
Amplitude: 5
Period: $\frac{\pi}{6}$
Phase Shift: right $\frac{\pi}{24}$
Vertical Shift: up 8

$y = 5 \sin \left( 12\left( x - \frac{\pi}{24}\right) \right) + 8$

21.
Amplitude: 1
Period: $\frac{\pi}{6}$
Phase Shift: left $\frac{6\pi}{7}$
Vertical Shift: up 2

$y = \sin \left( \frac{5}{3}(x + \frac{6\pi}{7}) \right) + 2$

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For 22–24, write the equation of the following sine curves. Use a positive leading coefficient $a$ and the closest phase shift possible (left or right). For some problems, it may be equal to move left or right.

22.
\[ y = 2 \sin \left( \frac{4}{3}(x - \frac{\pi}{6}) \right) - 1 \]

23.
\[ y = 3 \sin \left( \frac{4}{3}(x + \frac{\pi}{4}) \right) \]

24.
\[ y = \sin \left( \frac{4}{3}(x - \frac{\pi}{3}) + 1 \right) \]
For 25-27, write a sine AND cosine function for the curve. Use a positive leading coefficient $\alpha$ and the closest phase shift possible (left or right). For some problems, it may equal to move left or right.

25. sine: $y = 3\sin(\theta + \pi)$
cosine: $y = 3\cos(\theta + \pi)$

26. sine: $y = 2\sin\left(\frac{1}{2}\theta + \frac{3\pi}{8}\right)$
cosine: $y = 2\cos\left(\frac{1}{2}\theta - \frac{3\pi}{8}\right)$

27. sine: $y = \sin(4\theta - \frac{\pi}{2}) + 2$
cosine: $y = \cos(4\theta - \pi) + 2$

**Skillz Review:** Complex Fraction (Fraction in a fraction)

\[
\frac{\left(\frac{2}{5}\right)}{\left(\frac{3}{10}\right)} = \frac{2}{5} \cdot \frac{10}{3} = \frac{4}{3}
\]

\[
\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{1}{\cos x}\right)} = \frac{\cos x}{\sin x} = \cot x
\]

\[
csc \theta = \frac{1}{\sin \theta} = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{\sin^2 \theta}{\sin \theta} - \frac{1}{\sin \theta} = \frac{\sin^2 \theta - 1}{\sin \theta}
\]

**Skillz Review:** Add/Subtract Fractions

\[
\frac{3}{5} + \frac{1}{2} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}
\]

\[
\frac{1}{\cos x} + \frac{1}{5} = \frac{5 + \cos x}{5 \cos x}
\]