11.3 Sum and Difference Identities

Is it true?
$$\sin(45° + 30°) = \sin 45° + \sin 30°$$

Sum/Difference Identities
$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$$
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Ex 1:

Ex 2:

Ex 3:
Ex 4: Write the expression as the sine, cosine, or tangent of an angle.

Ex 5: Find $\sin(x - y)$ given the following:

Ex 6: Is the equation an identity?

SUMMARY:
### 11.3 Sum and Difference Identities

#### Directions: Tell whether each statement is true or false.

<table>
<thead>
<tr>
<th>1) ( \sin 75^\circ = \sin 50^\circ \cos 25^\circ - \cos 25^\circ \sin 25^\circ )</th>
<th>2) ( \cos 15^\circ = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ )</th>
<th>3) ( \tan 225^\circ = \frac{\tan 180^\circ - \tan 45^\circ}{1 + \tan 180^\circ \tan 45^\circ} )</th>
</tr>
</thead>
</table>

#### Directions: Write the expression as the sine, cosine or tangent of an angle.

<table>
<thead>
<tr>
<th>4) ( \sin 42^\circ \cos 17^\circ - \cos 42^\circ \sin 17^\circ )</th>
<th>5) ( \frac{\tan 19^\circ + \tan 47^\circ}{1 - \tan 19^\circ \tan 47^\circ} )</th>
<th>6) ( \cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4} )</th>
</tr>
</thead>
</table>

#### Directions: Use the sum or difference identity to find the exact value.

| 7) \( \tan 195^\circ \) | 8) \( \cos 255^\circ \) | 9) \( \sin 165^\circ \) | 10) \( \cos \frac{13\pi}{12} \) |
Directions: Find the exact value.

13) \sin(\alpha - \beta) 
\text{Given: } \cos \alpha = \frac{3}{5}, \text{ where } 0 < \alpha < \frac{\pi}{2} 
\tan \beta = \frac{12}{5}, \text{ where } 0 < \beta < \frac{\pi}{2} 

14) \tan(x - y) 
\text{Given: } \cos x = \frac{7}{25}, \text{ where } 0^\circ < x < 90^\circ 
\cos y = -\frac{4}{5}, \text{ where } 90^\circ < y < 180^\circ 

15) \sin(\alpha + \beta) 
\text{Given: } \sin \alpha = \frac{4}{5}, \text{ where } \alpha \text{ is in Quadrant I} 
\cos \beta = -\frac{24}{25}, \text{ where } \beta \text{ is in Quadrant III} 

16) \cos(x + y) 
\text{Given: } \cos x = \frac{15}{17}, \text{ where } \frac{3\pi}{2} < x < 2\pi 
\tan y = \frac{4}{3}, \text{ where } \pi < y < \frac{3\pi}{2}
### Directions: Is the equation an identity? Explain using the sum or difference identities

<table>
<thead>
<tr>
<th>Equation</th>
<th>Identity Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>17) ( \cos(x - \pi) = \cos x )</td>
<td>Use the identity: ( \cos(x - \pi) = -\cos x )</td>
</tr>
<tr>
<td>18) ( \sin(x - \pi) = \sin x )</td>
<td>Use the identity: ( \sin(x - \pi) = -\sin x )</td>
</tr>
</tbody>
</table>

### REVIEW SKILLZ: Directions: Solve each triangle.

1) [Diagram of triangle with sides 12.9 and 6.2, angle A not specified.]

2) [Diagram of triangle with sides 10 and side 67°, angle A not specified.]

### 11.3 Application and Extension

1) Find the exact value.
\( \cos 285^\circ \)

2) Find the exact value.
\( \cos(x + y) \)

*Given:* \( \cos x = \frac{15}{17}, \) \( \frac{3\pi}{2} < x < 2\pi \)

*Given:* \( \tan y = \frac{4}{3}, \) \( \pi < y < \frac{3\pi}{2} \)
3) Verify the following DOUBLE ANGLE IDENTITIES. (Hint: \( \sin(2x) = \sin(x + x) \))

a) \( \sin(2x) = 2 \sin x \cos x \)

b) \( \cos(2x) = 2 \cos^2 x - 1 \)

5) When a wave travels through a taut string (like a guitar string), the displacement \( y \) of each point on the string depends on the time \( t \) and the point's position \( x \). The equation of a standing wave can be obtained by adding the displacements of two waves traveling in opposite directions. Suppose two waves can be modeled by the following equations:

\[
y_1 = A \cos\left(\frac{2\pi t}{3} - \frac{2\pi x}{5}\right) \quad y_2 = A \cos\left(\frac{2\pi t}{3} + \frac{2\pi x}{5}\right)
\]

Find \( y_1 + y_2 \)

6) Mr. Sullivan has been carrying the other Algebros on his back for the last several years. He knows from Mr. Rahn's physics' class that the force \( F \) (in pounds) on a person's back when he bends over at an angle \( \theta \) is:

\[
F = \frac{0.6W \sin(\theta+90^\circ)}{\sin 12^\circ}
\]

Simplify the above formula.