5.2 Poly Division and Factor Theorem

Long Division:
1. \((2x^4 + 3x - 1) \div (x^2 + 2x + 1)\)
2. \((4x^3 + 30x^2 + 74x + 64) \div (2x + 7)\)

Synthetic Substitution / Division:
3. \(f(x) = 2x^3 - 5x^2 + 10\) \(f(3) = \)
4. \(f(x) = -5x^4 + x^3 - 3x\) \(f(-2) = \)

When can we use synthetic division?
5. \((2n^4 - 10n^3 + 30n + 28) \div (n - 3)\)
6. \((2x^2 - 5x - 12) \div (2x + 3)\)

Remainder Theorem:
If a polynomial \(f(x)\) is divided by \(x - k\), then the remainder \(r\) is

Factor Theorem:
\((x - k)\) is a factor of the polynomial \(f(x)\) if and only if

Given one zero of the polynomial function, find the other zeros.
7. \(f(x) = 2x^3 - 20x^2 + 66x - 72;\) \(3\) is a zero
8. \(f(x) = 2x^3 - 17x^2 + 90x - 41;\) \(\frac{1}{2}\) is a zero.
5.2 Poly Division and Factor Theorem

9. If \( x = 3 \) is a zero of \( f(x) \), what else can we conclude?

10. Given the graph of \( f(x) = 3x^4 - 8x^3 - 12x^2 + 24x + 9 \) to the right, use your knowledge of the factor theorem to find the exact values of all zeros of the function. List them from smallest to largest.

The Fundamental Theorem of Algebra:

If \( f(x) \) is a polynomial of degree \( n \), then \( f(x) = 0 \) has exactly \( n \) solutions (both real and imaginary).

Imaginary zeros will come in ____________________! In other words, each imaginary zero has a conjugate “buddy”, so there won’t ever be just one.

11. If \( 3 + 2i \) is a zero of a function, name one other zero.

12. If \( 7i - \sqrt{5} \) is a zero of a function, name one other zero.

**Skillz Review:** Find the \( x \)- and \( y \)-intercepts for each function. SHOW ALL WORK!

1. \( 12x - 5y = 60 \)
   - \( x \)-int: 
   - \( y \)-int: 

2. \( f(x) = \frac{x^2 + x - 6}{4x - 4} \)
   - \( x \)-int: 
   - \( y \)-int: 

Now summarize what you learned!
<table>
<thead>
<tr>
<th>For 1-3, use long division to divide the polynomials.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((2x^2 - 7x + 4) \div (x - 2))</td>
</tr>
<tr>
<td>2. ((y^2 - 9) \div (y + 1))</td>
</tr>
<tr>
<td>3. ((9x^5 - 3x^3 + 21x^2 - 2x + 4) \div (3x^2 + 1))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For 4-6, use synthetic division to divide the polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. ((x^2 + 3x - 3) \div (x - 3))</td>
</tr>
<tr>
<td>5. ((7x^6 + 50x^3 + 20x) \div (x + 2))</td>
</tr>
<tr>
<td>6. ((3x^4 - 4x^2 + 1) \div (x + 1))</td>
</tr>
</tbody>
</table>

7. If \((x + 1)\) is a factor of \(2x^5 + 2x^4 - 5x^3 - 5x^2 - 3x - 3\), what are all the factors of \(f(x)\).  
8. Is \((x - 11)\) a factor of \((3x^4 - 33x^3 - 17x^2 + 187x - 11)\)?
9. If \((3x + 5)\) is a factor of \(6x^3 + 31x^2 + 23x - 20\), what are all the factors of \(f(x)\).

10. Is \((n + 1)\) a factor of \((6n^3 + 6n^2)\)?

| For 11-14, a zero of the function is given. Find ALL the zeros of the function. |
|-----------------------------|-----------------------------|
| 11. \(f(x) = 4x^3 - 25x^2 - 154x + 40\); \(f(10) = 0\) | 12. \(f(x) = 3x^3 + 34x^2 + 72x - 64\); \(f(-4) = 0\) |
| 13. \(f(x) = x^5 + 2x^4 + 7x^3 + 14x^2 + 6x + 12\); \(f(-2) = 0\) | 14. \(f(x) = x^3 - 125\); \(f(5) = 0\) |
For 15-16, use the graph of the function to determine at least one zero, then find the exact values of all the zeros using the Factor Theorem.

15. \( f(x) = 3x^4 + 16x^3 - 8x^2 - 112x - 91 \)

![Graph of the function for 15](image1)

16. \( f(x) = 10x^3 - 31x^2 - 76x + 160 \)

![Graph of the function for 16](image2)

For 17-20, one zero is given of \( f(x) \). List one other zero.

17. \( 13 - 25i \)  
18. \( 10i \)  
19. \( 3i + 1 \)  
20. \( \sqrt{7} + 14i \)
5.2 Application and Extension

1. Use long division. \((a^3 + 5a^2 + 10a + 2) ÷ (a + 2)\)

2. Is \((k + 8)\) a factor of \((k^4 - 50k^2 - 16k + 1)\)?

3. During Mr. Brust’s annual visit to the scrapbooking convention, he took a 20 by 40 inch piece of cardboard and put pretty flowers on it. He then realized it was possible to make a box with a hinged lid by cutting out six squares, \(x\) inches on a side, from each corner and the middle, and then the ends and sides will be folded up to form the box and its lid (see figure). We want the box to have a volume of 625 cubic inches. One possible solution would be if the cutout was 5 inches. What other value of \(x\) would result in a box with a volume of 625 cubic inches. *(Hint: you will need to use the quadratic formula at some point.)*