### 6.4 Variation and Modeling

**Directions:** Write the equation of variation for each situation, use \( k \) as the constant of variation.

1) \( F \) is inversely proportional to \( x \)
\[
F = \frac{k}{x}
\]

2) \( R \) is jointly proportional to \( S \) and \( T \).
\[
R = k ST
\]

3) \( R \) varies directly as \( m \) and inversely as the square of \( d \).
\[
R = \frac{km}{d^2}
\]

4) Kinetic energy, \( E \), is directly proportional to the square of the velocity, \( v \) and the mass \( m \).
\[
E = k \cdot v^2 m
\]

**Directions:** Write the equation of variation for each situation and solve.

5) \( U \) varies directly as the square root of \( v \). If \( u = 3 \) when \( v = 4 \), find \( u \) when \( v = 10 \).
\[
\begin{align*}
U &= k \cdot \sqrt{v} \\
\frac{u}{\sqrt{v}} &= k \\
U &= \frac{2 \cdot \sqrt{10}}{3} \\
U &= \frac{2 \cdot 3.16}{3} \\
U &= \frac{6.32}{3} \\
U &= 2.11
\end{align*}
\]

6) \( Y \) varies directly as the cube of \( x \). If \( y = 48 \) when \( x = 4 \), find \( y \) when \( x = 8 \).
\[
\begin{align*}
y &= k \cdot x^3 \\
\frac{y}{x^3} &= k \\
y &= \frac{3}{4} \cdot (8)^3 \\
y &= 384
\end{align*}
\]

7) \( Q \) varies jointly as \( m \) and the square of \( n \), and inversely as \( P \). If \( Q = 2 \) when \( m = 3 \), \( n = 6 \), and \( P = 12 \), find \( Q \) when \( m = 4 \), \( n = 18 \), and \( P = 2 \).
\[
\begin{align*}
Q &= \frac{k \cdot m \cdot n^2}{P} \\
\frac{2}{P} &= \frac{k \cdot 3 \cdot 6^2}{P} \\
2 &= 27 \cdot 36 \\
\frac{1}{P} &= \frac{1}{36} \\
\frac{1}{36} &= \frac{1}{12} \\
36 &= 192 \cdot k \\
\frac{1}{12} &= \frac{3}{16} \\
Q &= 144
\end{align*}
\]

8) \( W \) varies jointly as \( x \), \( y \) and \( z \). If \( w = 36 \) when \( x = 2 \), \( y = 8 \), and \( z = 12 \), find \( w \) when \( x = 1 \), \( y = 2 \), and \( z = 4 \).
\[
\begin{align*}
w &= k \cdot x \cdot y \cdot z \\
\frac{36}{xyz} &= k \\
\frac{36}{2 \cdot 8 \cdot 12} &= k \\
\frac{3}{16} &= k \\
w &= \frac{3}{16} \cdot (1) \cdot (2) \cdot (4) \\
w &= \frac{3}{2}
\end{align*}
\]

**Directions:** Translate each statement into an equation using \( k \) as the constant of variation.

9) The length of time, \( t \), that it takes fruit to ripen is inversely proportional to the sum, \( T \), of the average daily temperatures during the growing season.
\[
t = \frac{k}{T}
\]

10) The maximum safe load, \( L \), for a horizontal beam varies jointly as its width, \( w \), and the square of its height, \( h \), and inversely as its length, \( x \).
\[
L = \frac{k \cdot w \cdot h^2}{x}
\]
11) The number, \( N \), of long-distance phone calls between two cities varies jointly as the populations \( P_1 \) and \( P_2 \) of the two cities, and inversely as the distance, \( d \), between the two cities.  
\[
N = \frac{k \cdot P_1 \cdot P_2}{d}
\]

12) The erosive force, \( P \), of a swiftly flowing stream is directly proportional to the sixth power of the velocity, \( v \), of the water.  
\[
P = k \cdot v^6
\]

Directions: Write the equation of variation for each situation and solve.

13) The weight, \( w \), of an object on or above the surface of the Earth varies inversely as the distance, \( d \), between the object and the center of the Earth. If a girl weighs 100 pounds on the surface of the Earth, how much would she weigh 400 miles above Earth’s surface? Assume the radius of the Earth is 4,000 miles.  
\[
w = \frac{k}{d}
\]
\[
100 = \frac{k}{4000}
\]
\[
w = \frac{400,000}{4,400}
\]
\[
w = 90.9 \text{ pounds}
\]

14) Ohm’s Law states that the current, \( I \), in a wire varies directly as the electromotive forces, \( E \), and inversely as the resistance, \( R \). If \( I = 22 \) amperes when \( E = 110 \) volts and \( R = 5 \) ohms, find \( I \) if \( E = 220 \) volts and \( R = 11 \) ohms.  
\[
I = \frac{E}{R}
\]
\[
22 = \frac{110}{5}
\]
\[
I = \frac{220}{11}
\]
\[
I = 20 \text{ amperes}
\]

15) If the amount of time, \( t \), it takes Sully to complete one unit of Pre-Calc varies jointly as the number of sections, \( s \), and the number of mastery checks per section, \( m \), and inversely as the square root of the number of problems per section, \( p \), and \( t = 12 \) when \( s = 3 \), \( m = 2 \), and \( p = 64 \), find \( t \) when \( s = 5 \), \( m = 2 \), and \( p = 25 \).  
\[
t = \frac{k \cdot s \cdot m}{\sqrt{p}}
\]
\[
t = \frac{16 \cdot 5 \cdot 2}{\sqrt{25}}
\]
\[
t = 32 \text{ hrs}
\]

16) The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. A wire with a length of 200 inches and a diameter of one-quarter of an inch has a resistance of 20 ohms. Find the electrical resistance in a 500 inch wire with the same diameter.  
\[
R = \frac{k \cdot l}{d^2}
\]
\[
20 = \frac{k \cdot 200}{\left(\frac{1}{4}\right)^2}
\]
\[
0.0625k = R
\]