Unit 3 Corrective Assignment – Function Analysis

Pre-Calculus

1. \( h(t) = \sqrt{t + 49} \geq 0 \)
   \[ t \geq -49, \quad t \neq 7 \]
   Domain: \( (-\infty, -2] \cup [2, \infty) \)
   Abs max/min value(s): none
   Increasing: \( (-\infty, -2) \)
   Decreasing: \( (2, \infty) \)
   Left End-behavior: \( \lim_{x \to -\infty} f(x) = -\infty \)
   Right End-behavior: \( \lim_{x \to \infty} f(x) = \infty \)

2. \( f(x) = \frac{8 + x}{64 + x^2} \neq 0 \)
   \( x^2 \neq -64 \)
   \( x \neq \pm 8i \)
   Domain: \( \mathbb{R} \)
   W\#0, W\#3, W\# \frac{3}{2} \)
   holes V.A.

3. \( g(w) = \frac{w^2 - 3w}{2w^3 + w^2 - 21w} \)

4. \( f(x) = \frac{x^2 - 9}{x - 3} \)
   Domain: \( (-\infty, 3) \cup (3, \infty) \)
   Absolute max/min value(s): none
   Local extrema that are NOT absolute: none
   Increasing: \( (3, \infty) \)
   Decreasing: \( (-\infty, 3) \)
   Left End-behavior: \( \lim_{x \to -\infty} f(x) = \infty \)
   Right End-behavior: \( \lim_{x \to \infty} f(x) = \infty \)
Find the value of the given function at the indicated domain value.

\[ g(x) = \begin{cases} 
  x^2 + 7x - 5, & x < -5 \\
  5 - x^3, & -3 \leq x < 5 \\
  -\sqrt{x} - 9, & x \geq 5 
\end{cases} \quad h(x) = \begin{cases} 
  -x^2 - 4x + 6, & x < -2 \\
  \frac{2}{3}x - 5, & -2 \leq x < 5 \\
  |x - 15| - 2, & x \geq 5 
\end{cases} \]

6. \( g(-4) = \) DNE

7. \( g(5) = \) DNE

8. \( h(10) = \) 3

9. \( h(-2) = \) DNE

10. \( h(5) = \) 8

11. \( h(3) = \) -3

12. \( g(9) = \) 0

13. \( h(-3) = \) 9

Graph the following piecewise functions.

\[ f(x) = \begin{cases} 
  -\frac{1}{3}x - 2, & x < -3 \\
  2, & -3 \leq x \leq 2 \\
  x - 5, & x > 2 
\end{cases} \]

\[ g(x) = \begin{cases} 
  -3, & -4 \leq x < -3 \\
  1, & -3 \leq x \leq 0 \\
  -|x - 2|, & x > 0 
\end{cases} \]

\[ h(x) = \begin{cases} 
  -2(x + 4)^2 + 5, & x \leq -2 \\
  -\frac{1}{2}x, & x > 2 
\end{cases} \]

Skills Review: Solve or evaluate.

17. \( \sqrt{-245} = 7i\sqrt{5} \)

18. \( 7x^2 + 8 = 358 \)

19. \( -3(x + 2)^2 - 1 = -49 \)

20. \( 2(x + 6)^2 = -68 \)

\( x = \pm 5\sqrt{2} \) or \( x = -6 \) or \( x = -6 \pm i\sqrt{34} \)
Given the graph of \( f \), write out the function’s equation. Use a linear expression \((mx + b)\) for straight lines, absolute values if there is a “V” graph.

21. \[ f(x) = \begin{cases} 
2x + 7, & x \leq -2 \\
4, & -2 < x \leq 3 \\
-x, & x > 3 
\end{cases} \]

22. \[ f(x) = \begin{cases} 
-(x+4)^2 - 1, & x < -2 \\
-1, & -2 \leq x < 2 \\
1, & x = 2 \\
-x + 5, & x > 2 
\end{cases} \]

23. Is this function continuous? (SHOW WORK!)
\[ f(x) = \begin{cases} 
20 - 3x, & x < 8 \\
-\sqrt{x - 4}, & x \geq 8 
\end{cases} \]
\[
20 - 3(8) = -\sqrt{(8) - 4} \\
-4 = -\sqrt{4} \\
-4 = -2 \quad \text{No.}
\]

24. What value(s) of \( k \) would make the function continuous?
\[ h(x) = \begin{cases} 
\sqrt{13 - x}, & x \leq 87 \\
k^2 - 3k, & x > 87 
\end{cases} \]
\[
\sqrt{13 - (-87)} = k^2 - 3k \\
10 = k^2 - 3k \\
0 = k^2 - 3k - 10 \\
0 = (k - 5)(k + 2) \\
k = 5 \text{ or } k = -2
\]

25. Mr. Kelly wants to create a rectangular feeding pen for his chickens, but only has 70 meters of fencing. He decides to use the side of his house as one side of the pen.

a. If \( x \) represents the width of the pen, express its area \( A \) in terms of \( x \). (The side of Kelly’s house is the length.)
\[ A(x) = 70x - 2x^2 \]

b. What is the domain of the function \( A \) (determined by the physical restrictions)?
\[ 0 < x < 35 \]

26. Rewrite the function \( f(x) = \frac{1}{3}|x - 15| - 8 \) as a piecewise function.
\[ f(x) = \begin{cases} 
-\frac{1}{3}x - 3, & x \leq 15 \\
\frac{1}{3}x - 13, & x > 15 
\end{cases} \]
27. A rectangle has its base on the $x$-axis and its two upper corners on the parabola $y = 4 - x^2$.

a. Draw this scenario on the coordinate plane to the right, and draw one possible rectangle.

b. Label the base and height of your rectangle in terms of $x$.

$$b = 2x, \quad h = 4 - x^2$$

c. Find the function $A(x)$ that represents the area of the rectangle.

$$A(x) = 8x - 2x^3$$

d. What is the largest possible area of this rectangle?

$$6.158$$

e. At what $x$-value should the rectangle be drawn for the largest area?

$$1.155$$

28. Kelly is headed off to Hickville, New York and is renting a car to get there from Sully's house in O-high-O. He needs to rent a car to get there and finds one car rental agency that charges $0.21$ per mile if the total mileage does not exceed 75. If the total mileage exceeds 75, the agency charges $0.21$ per mile for the first 75 miles and only $0.16$ per mile for each mile over 75. If $m$ represents the number of miles a rented vehicle is driven, express the mileage charge $C(m)$ as a function of $m$. Find $C(24)$ and $C(205)$.

$$0.21(75) = 0.16(75) + b$$
$$15.75 = 12 + b$$
$$3.75 = b$$

$$C(m) = \begin{cases} 
0.21m, & m \leq 75 \\
0.16m + 3.75, & m > 75 
\end{cases}$$

$$C(24) = 5.04$$

$$C(205) = 36.55$$