

14.3 The Binomial Theorem

3

Practice 14.3

Evaluate each combination. Use the formula and then check your answers with your calculator.

1. ${}_{11}C_5 = 462$

$$= \frac{11!}{5!6!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{5 \cdot 4 \cdot 3 \cdot 2 \cdot \cancel{6!}}$$

$$= 11 \cdot 2 \cdot 3 \cdot 7$$

 Find each term described.

2. ${}_{13}C_8 = 1287$

$$\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{5 \cdot 4 \cdot 3 \cdot 2 \cdot \cancel{8!}}$$

$$13 \cdot 11 \cdot 9 = 1287$$

3. ${}_{10}C_6 = 210$

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{4 \cdot 3 \cdot 2 \cdot \cancel{6!}}$$

$$= 10 \cdot 3 \cdot 7$$

4. ${}_{10}C_4 = 210$
 SAME AS
 #3

5) 4th term in expansion of $(2x - 1)^3$

$$\binom{3}{3}(2x)^0(-1)^3$$

$$= 1 \cdot 1 \cdot -1 = \boxed{-1}$$

6) 1st term in expansion of $(4x - 2y)^4$

$$\binom{4}{0}(4x)^4(-2y)^{4-4}$$

$$256x^4$$

$$\boxed{= 256x^4}$$

7) 3rd term in expansion of $(y + 3)^4$

$$\binom{4}{2}(y)^2(3)^2$$

$$6y^2 \cdot 9$$

$$\boxed{= 54y^2}$$

8) 2nd term in expansion of $(2u + v)^4$

$$\binom{4}{1}(2u)^3(v)^1$$

$$4 \cdot 8u^3 \cdot v$$

$$\boxed{= 32u^3v}$$

 Expand completely.

9) 5th term in expansion of $(x + 2y)^4$

$$\binom{4}{4}(x)^0(2y)^4$$

$$1 \cdot 1 \cdot 16y^4$$

$$\boxed{= 16y^4}$$

10) 3rd term in expansion of $(y - 5x)^3$

$$\binom{3}{2}(y)^1(-5x)^2$$

$$3 \cdot 1 \cdot -125x^2$$

$$3 \cdot -125x^2y$$

$$\boxed{-375x^2y}$$

 13) $(x - 4y)^4$

11) $(2n - 1)^3$

$$1 \binom{3}{0}(2n)^3(-1)^0 + 3 \binom{3}{1}(2n)^2(-1)^1 + 3 \binom{3}{2}(2n)^1(-1)^2 + 1 \binom{3}{3}(2n)^0(-1)^3$$

$$\boxed{= 8n^3 - 12n^2 + 6n - 1}$$

12) $(1 + 3y)^3$

$$1 \binom{3}{0}(3y)^0 + 3 \binom{3}{1}(3y)^1 + 3 \binom{3}{2}(3y)^2 + 1 \binom{3}{3}(3y)^3$$

$$= 1 + 9y + 27y^2 + 27y^3$$

$$1 \binom{4}{0}(x)^4(-4y)^0 + 4 \binom{4}{1}(x)^3(-4y)^1 + 6 \binom{4}{2}(x)^2(-4y)^2 + 4 \binom{4}{3}(x)^1(-4y)^3 + 1 \binom{4}{4}(x)^0(-4y)^4$$

$$x^4 - 16x^3y + 96x^2y^2 - 256xy^3 + 256y^4$$

14) $(x + 4)^4$

$$1 \binom{4}{0}(x)^4(4)^0 + 4 \binom{4}{1}(x)^3(4)^1 + 6 \binom{4}{2}(x)^2(4)^2 + 4 \binom{4}{3}(x)^1(4)^3 + 1 \binom{4}{4}(x)^0(4)^4$$

$$= x^4 + 16x^3 + 96x^2 + 256x + 256$$

15) $(y - 3x)^5$

$$1 \binom{5}{0}(y)^5(-3x)^0 + 5 \binom{5}{1}(y)^4(-3x)^1 + 10 \binom{5}{2}(y)^3(-3x)^2 + 10 \binom{5}{3}(y)^2(-3x)^3 + 5 \binom{5}{4}(y)^1(-3x)^4 + 1 \binom{5}{5}(y)^0(-3x)^5$$

$$= y^5 - 15y^4x + 90y^3x^2 - 270y^2x^3 + 405yx^4 - 243x^5$$