

2.12 Logarithmic Function Manipulation

2.12 Practice

AP Precalculus

Instructions: Let x and y be positive constants. Write each expression as a sum or difference of logarithms.

1. $\log_3(xy^3)$

$$\log_3 x + \log_3 y^3$$

$$\log_3 x + 3 \log_3 y$$

2. $\ln \frac{x^2}{y^3}$

$$\ln x^2 - \ln y^3$$

$$2 \ln x - 3 \ln y$$

3. $\log 1000x^3$

$$\log 1000 + \log x^3$$

$$3 + 3 \log x$$

4. $\log_2(8\sqrt{x})$

$$\log_2 8 + \log_2 x^{1/2}$$

$$3 + \frac{1}{2} \log_2 x$$

5. $\log_2 \sqrt{\frac{16x^3}{y^2}} = \frac{1}{2} (\log_2 16x^3 - \log_2 y^2)$

$$\frac{1}{2} (\log_2 16 + \log_2 x^3 - 2 \log_2 y)$$

$$\frac{1}{2} (4 + 3 \log_2 x - 2 \log_2 y)$$

6. $\log_3(9x - 27)$

$$\log_3(9)(x-3)$$

$$\log_3 9 + \log_3(x-3)$$

$$2 + \log_3(x-3)$$

Instructions: Let x and y be positive constants. Write each as a single logarithm.

7. $\ln 3 + \ln x$

$$\ln 3x$$

8. $2 \log_3 x - 4 \log_3 y$

$$\log_3 x^2 - \log_3 y^4$$

$$\log_3 \frac{x^2}{y^4}$$

9. $\frac{1}{2}(\log x + 3 \log y)$

$$\frac{1}{2} (\log x + \log y^3)$$

$$\frac{1}{2} (\log xy^3)$$

$$\log (xy^3)^{1/2} = \log \sqrt{xy^3}$$

10. $3 \log_5(x - 4) - 2 \log_5 y$

$$\log_5 (x-4)^3 - \log_5 y^2$$

$$\log_5 \frac{(x-4)^3}{y^2}$$

CALCULATOR ACTIVE: Instructions: Use the change of base to change each to a logarithm with base 10 or base e . Then use a calculator and find the value of the logarithm to the nearest thousandth.

11. $\log_4 123$

$$\frac{\log 123}{\log 4} \quad \text{or} \quad \frac{\ln 123}{\ln 4}$$

3.471

12. $\log_9 578$

$$\frac{\log 578}{\log 9} \quad \text{or} \quad \frac{\ln 578}{\ln 9}$$

2.894

Instructions: Describe any transformations on the function, then find the domain and range of the transformed function.

13. $f(x) = \log_4 16x$

$$f(x) = \log_4 16 + \log_4 x$$

$$f(x) = 2 + \log_4 x$$

D: $(0, \infty)$ T: Vertical shift up 2 units
R: $(-\infty, \infty)$

14. $g(x) = \log_2 \left(\frac{x}{8}\right)$

$$g(x) = \log_2 x - \log_2 8$$

$$g(x) = \log_2 x - 3$$

D: $(0, \infty)$ T: Vertical translation Down 3
R: $(-\infty, \infty)$

15. $f(x) = \log_5 x^2$

$$f(x) = 2 \cdot \log_5 x$$

D = $(0, \infty)$
R = $(-\infty, \infty)$
T: Vertical Dilatation of 2.

16. $h(x) = \log_2(32 - 16x)$

$$h(x) = \log_2(16(2-x))$$

$$h(x) = \log_2 16 + \log_2(-(x-2))$$

$$h(x) = 4 + \log_2(-(x-2))$$

D = $(-\infty, 2)$ T: Vertical shift up 4
R = $(-\infty, \infty)$ Horizontal Reflection

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2.12 Test Prep

17. Let x and y be positive constants. Which of the following is equivalent to $5 \log x + 6 \log y$.

(A) $\log(x^5 + y^6)$

(B) $\log(x^5 y^6)$

(C) $\log(x^5 - y^6)$

(D) $\log(5x + 6y)$

$$\log x^5 + \log y^6$$

$$\log x^5 y^6$$

18. Let x and y be positive constants. Which of the following is NOT equivalent to:

$$\log_3(x+3) + 2\log_3(x-1) - \log_3(x+2).$$

(A) $\log_3\left(\frac{2(x+3)(x-1)}{(x+2)}\right)$

(B) $\log_3\left(\frac{(x+3)(x-1)(x-1)}{(x+2)}\right)$

(C) $\log_3\left(\frac{(x+3)(x-1)^2}{(x+2)}\right)$

(D) $\log_3\left(\frac{(x+3)(x^2-2x+1)}{(x+2)}\right)$

$$\log_3(x+3) + \log_3(x-1)^2 - \log_3(x+2)$$

$$\log_3 \frac{(x+3)(x-1)(x-1)}{x+2}$$

$$(x-1)(x-1)$$

$$\downarrow$$

$$x^2 - 2x + 1$$

19. If $\log 3 = x$ and $\log 4 = y$, which of the following expresses $\log \sqrt{12}$?

(A) $\frac{1}{2}(x-y)$

(B) $\frac{1}{2}(xy)$

(C) $\frac{1}{2}x + 2$

(D) $\frac{1}{2}(x+y)$

$$\log \sqrt{3 \cdot 4}$$

$$\log \sqrt{3} + \log \sqrt{4}$$

$$\log 3^{1/2} + \log 4^{1/2}$$

$$\frac{1}{2} \log 3 + \frac{1}{2} \log 4$$

$$\frac{1}{2} (\log 3 + \log 4)$$

$$\frac{1}{2} (x + y)$$

20. Approximate $\log_b 18$, given that $\log_b 2 \approx .4307$ and $\log_b 3 \approx .6827$.

(A) 0.8968

(B) 1.1134

(C) 1.5441

(D) 1.7961

$$\log_b 18 = \log_b(2 \cdot 9) = \log_b 2 \cdot 3^2$$

$$= \log_b 2 + 2 \log_b 3$$

$$= .4307 + 2(.6827)$$

$$= 1.7961$$