2.13B Exponential and Logarithmic Equations and Inequalities

AP Precalculus

2.13B Practice

CALCULATOR ACTIVE: Instructions: Solve each inequality.

1) $log_4 x < 3$

$\boxed{x > 0}$

$x < 4^3$

$x < 64$

so $\boxed{0 < x < 64 \text{ or } (0, 64)}$

2) $2^{x+3} + 5 > 37$

$2^{x+3} > 32$

$\log_2 2^{x+3} > \log_2 32$

$x + 3 > 5$

$\boxed{x > 2 \text{ or } (2, \infty)}$

3) $\log 5 + \log(x-2) \geq \log(3x+8)$

$x > 2$
$x > -\frac{8}{3}$

$\log(5(x-2)) \geq \log(3x+8)$

$5(x-2) \geq 3x+8$

$5x - 10 \geq 3x + 8$

$2x \geq 18$

$x \geq 9$

so... $\boxed{x \geq 9 \text{ or } (9, \infty)}$

4) $4(3^{2x}) - 8 \leq 316$

$4(3^{2x}) \leq 324$

$3^{2x} \leq 81$

$\log_3 3^{2x} \leq \log_3 81$

$2x \leq 4$

$\boxed{x \leq 2 \text{ or } (-\infty, 2)}$

5) $log_2(x+5) - 8 > -3$

$\boxed{x > -5}$

$\log_2(x+5) > 5$

$x + 5 > 2^5$

$x + 5 > 32$

$x > 27$

so... $\boxed{x > 27 \text{ or } (27, \infty)}$

6) $\ln(x+4) \leq \ln(x-6) + \ln 3$

$x > -4$
$x > 6$

$\ln(x+4) \leq \ln(3(x-6))$

$x + 4 \leq 3(x-6)$

$x + 4 \leq 3x - 18$

$22 \leq 2x$

$11 \leq x$

so $\boxed{x \geq 11 \text{ or } (11, \infty)}$

Instructions: Find the inverse of each function.

7) $g(x) = \ln(x-4) + 8$

$x = \ln(y-4) + 8$

$e^{x-8} = e^{\ln(y-4)}$

$e^{x-8} = y - 4$

$\boxed{e^{x-8} + 4 = y = g^{-1}(x)}$

8) $f(x) = 3(2^x) + 6$

$x = 3(2^y) + 6$

$x - 6 = 3(2^y)$

$\frac{x-6}{3} = 2^y$

$\log_2\left(\frac{x-6}{3}\right) = \log_2 2^y$

$\boxed{\log_2\left(\frac{x-6}{3}\right) = y}$

9) $h(x) = 3log_2(2x+1) - 3$

$x = 3\log_2(2y+1) - 3$

$x + 3 = 3\log_2(2y+1)$

$\frac{x+3}{3} = \log_2(2y+1)$

$2 \qquad 2$

$2^{\frac{x+3}{3}} = 2y + 1$

$2^{\frac{x+3}{3}} - 1 = 2y$

$\boxed{\frac{2^{\frac{x+3}{3}} - 1}{2} = y}$

10) $j(x) = 2e^{x+8} - 5$

$x = 2e^{y+8} - 5$

$x + 5 = 2e^{y+8}$

$\frac{x+5}{2} = e^{y+8}$

$\ln\left(\frac{x+5}{2}\right) = \ln e^{y+8}$

$\ln\left(\frac{x+5}{2}\right) = y + 8$

$\boxed{\ln\left(\frac{x+5}{2}\right) - 8 = y}$

CALCULATOR ACTIVE: Instructions: Solve.

11) Use the formula for continuously compounded to solve. $A = Pe^{rt}$, where A is how much money we currently have, P is the principal (how much we started with), r is the interest rate and t, is the amount of time in years.

If Mr. Brust currently has $250,000 in his retirement account that earns him 8.5%, how long will it take him to earn at least $1,000,000?

$1{,}000{,}000 < 250{,}000e^{0.085t}$

$4 < e^{0.085t}$

$\ln 4 < \ln e^{0.085t}$

$\ln 4 < 0.085t$

$\frac{\ln 4}{0.085} < t \longrightarrow \boxed{t > 16.3 \text{ years}}$

12) When considering the equation $\log(x-3)+\log(5)>\log(x+9)$, which of the following domains is our initial restriction.
   (A) $(3,\infty)$
   (B) $(5,\infty)$
   (C) $(-9,\infty)$
   (D) $(6,\infty)$

$x>3$ $\quad$ $x>-9$

↑ THIS DOMAIN IS THE INTERSECTION OF BOTH

13) When considering the equation $\log(x-3)+\log(5)>\log(x+9)$, which of the following represents the domain of all solutions to the inequality?
(A) $(3,\infty)$
(B) $(5,\infty)$
(C) $(-9,\infty)$
(D) $(6,\infty)$

$$\log(5(x-3))>\log(x+9)$$
$$5(x-3)>x+9$$
$$5x-15>x+9$$
$$4x>24$$
$$x>6$$

14) Express y as a function of x. A, B and C are constant, positive numbers.

$$\log(y-A)=Bx-\log C$$

(A) $y=\frac{10^{Bx}}{C}+A$
(B) $y=C+A(10^{Bx})$
(C) $y=\frac{C}{A}-10^{Bx}$
(D) $y=\frac{Bx^{10}}{C}+A$

$$\log(y-A)+\log C=Bx$$
$$\log(C(y-A))=Bx$$
$$C(y-A)=10^{Bx}$$
$$y-A=\frac{10^{Bx}}{C}$$
$$\boxed{y=\frac{10^{Bx}}{C}+A}$$