4.11 The Inverse and Determinant of a Matrix

AP Precalculus

4.11 Practice Solutions

Directions: Find the determinant of each matrix. Tell whether or not the matrix will have an inverse.

1) $\begin{bmatrix} 5 & 7 \\ -5 & 9 \end{bmatrix}$

$5(9) - (-5)(7)$

$45 + 35 = 80$

2) $\begin{bmatrix} -\frac{1}{2} & 3 \\ \frac{5}{6} & \frac{4}{3} \end{bmatrix}$

$(-\frac{1}{2})(\frac{4}{3}) - 3(\frac{5}{6})$

$-\frac{4}{6} - \frac{15}{6} = \boxed{-\frac{19}{6}}$

3) $\begin{bmatrix} -4.5 & -9 \\ -3 & -6 \end{bmatrix}$

$-4.5(-6) - (-3)(-9)$

$27 - 27$

0

Directions: Find the inverse of each matrix if possible. If it's not possible, explain, why it isn't.

4) $\begin{bmatrix} 2 & 8 \\ 0 & 3 \end{bmatrix}$ $2(3) - 8(0)$ $6 - 0 = 6$

$\frac{1}{6}\begin{bmatrix} 3 & -8 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{6} & -\frac{8}{6} \\ 0 & \frac{2}{6} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & -\frac{4}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$

5) $\begin{bmatrix} -3 & -6 \\ -2 & 4 \end{bmatrix}$ $-12 - 12 = -24$

$\frac{1}{-24}\begin{bmatrix} 4 & 6 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{4}{24} & -\frac{6}{24} \\ \frac{2}{24} & -\frac{3}{24} \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{6} & -\frac{1}{4} \\ -\frac{1}{12} & \frac{1}{8} \end{bmatrix}$

6) $\begin{bmatrix} -5 & 4 \\ 8 & -8 \end{bmatrix}$ $40 - 32 = 8$

$\frac{1}{8}\begin{bmatrix} -8 & -4 \\ -8 & -5 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{4}{8} \\ -1 & -\frac{5}{8} \end{bmatrix}$

$= \begin{bmatrix} -1 & -\frac{1}{2} \\ -1 & -\frac{5}{8} \end{bmatrix}$

CALCULATOR ACTIVE: Directions: Multiply the matrices and determine if they are inverses.

7) $\begin{bmatrix} 5 & 8 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ \frac{3}{4} & -\frac{5}{4} \end{bmatrix}$

NORMAL FLOAT AUTO REAL DEGREE MP

[A]*[B]

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

IDENTITY MATRIX

INVERSES

8) $\begin{bmatrix} -\frac{17}{18} & -\frac{5}{18} & \frac{2}{9} \\ \frac{23}{9} & \frac{11}{9} & -\frac{7}{9} \\ -\frac{11}{6} & -\frac{5}{6} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 & 1 \\ 5 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$

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[C]*[D]

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

INVERSES

9) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & -2 \\ 2 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

NORMAL FLOAT AUTO REAL DEGREE MP

[E]*[F]

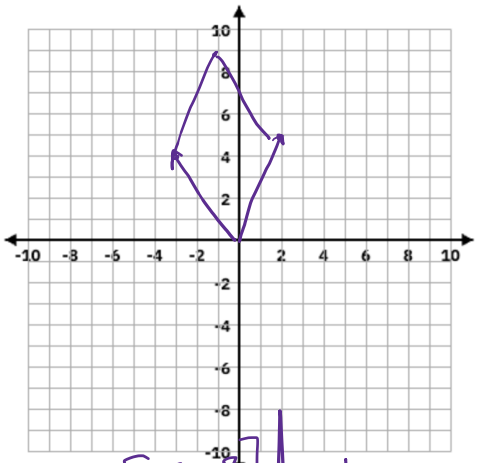
$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 2 & -1 & -2 \end{bmatrix}$

NOT IDENTITY MATRIX

NOT INVERSES

Directions: Plot the parallelogram formed by the vectors and then find the area.

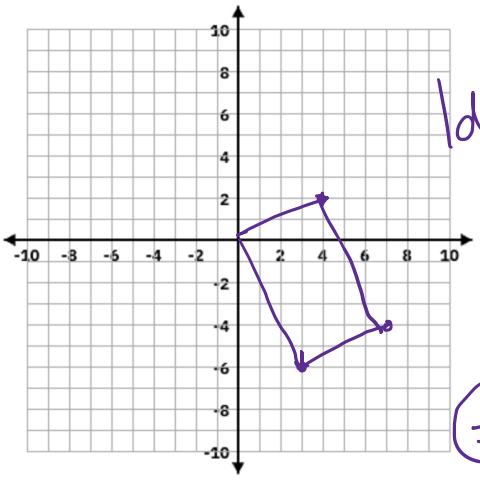
10) $\langle -3, 4 \rangle$ and $\langle 2, 5 \rangle$



$\left|\det\begin{bmatrix} -3 & 2 \\ 4 & 5 \end{bmatrix}\right| = |-15 - 8|$

$= |-23| = 23u^2$

11) $\langle 4, 2 \rangle$ *and* $\langle 3, -6 \rangle$



$\left|\det\begin{bmatrix} 4 & 3 \\ 2 & -6 \end{bmatrix}\right|$

$|-24 - 6|$

$|-30|$

$30u^2$

12) (2.3)

x	0	1	2	3	4
$f(x)$	40 $\times\frac{1}{2}$	20 $\times\frac{1}{2}$	10 $\times\frac{1}{2}$	5 $\times\frac{1}{2}$	$\frac{5}{2}$

The exponential function f is defined by $f(x) = ab^x$, where a and b are positive constants. The table gives values of $f(x)$ at selected values of x. Which of the following statements is true?

(A) f demonstrates exponential decay because $a > 0$ and $0 < b < 1$.

(B) f demonstrates exponential decay because $a > 0$ and $b > 1$.

(C) f demonstrates exponential growth because $a > 0$ and $0 < b < 1$.

(D) f demonstrates exponential growth because $a > 0$ and $b > 1$.

13) (2.5A) **Calculator active.** In the xy-plane, the graphs of the linear function L and the exponential function E both pass through the points $(0, 4)$ and $(1, 8)$. The exponential function E is in the form $E(x) = ab^x$. The function f is given by $f(x) = E(x) - L(x)$. What is the minimum value of f.

$L(x) = m(x - x_1) + y_1$

$m = \frac{8-4}{1-0} = 4$

$L(x) = 4(x - 0) + 4$

$L(x) = 4x + 4$

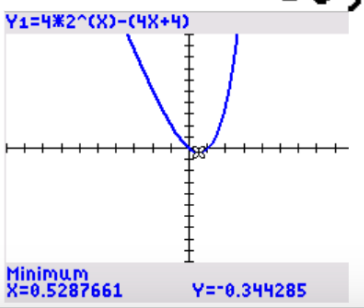
$f(x) = E(x) - L(x)$

$E(x) = a \cdot b^x$

$b = 2$ ← multiply 4 by 2 to get 8.

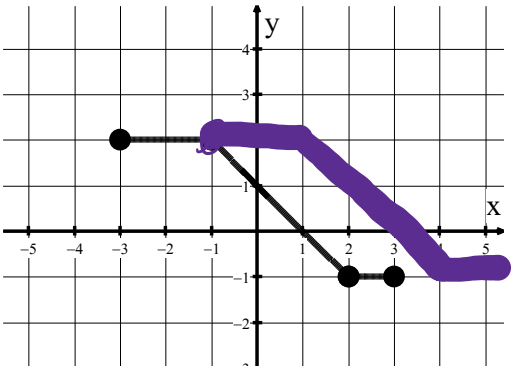
$a = 4$ ← initial value

$E(x) = 4 \cdot 2^x$



Remember, a value of f is always a-value.

13) (2.7B) The piecewise-linear function f, defined on $-3 \leq x \leq 3$, is shown in the graph. The function g is given by $g(x) = x - 2$. Sketch a graph of $y = f(g(x))$.



Graph of *f*

$y = f(x-2)$

shift right 2 units