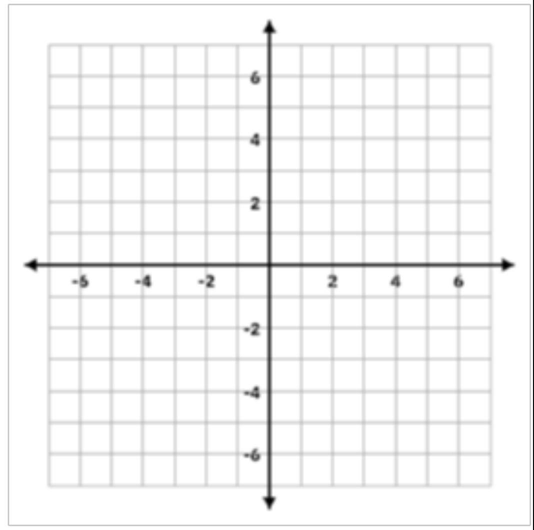
4.12 Linear Transformations and Matrices

AP Precalculus Name: ______________________________

CA #1

Directions: Apply the transformation for each matrix *A*.

1) Apply the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ if $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$.



a. If $\vec{u} = \langle -3,2 \rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

b. If $\vec{v} = \langle 1,3 \rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

c. Describe the transformation that occurs.

d. What is the general transformation that occurs to $\langle x, y \rangle$.

Directions: Determine if T: $\mathbb{R}^2 \to \mathbb{R}^2$ given by T $(\langle x, y \rangle)$ is a linear transformation.

2) **T $(\langle x, y \rangle) \to \langle 2x, x - y \rangle$**

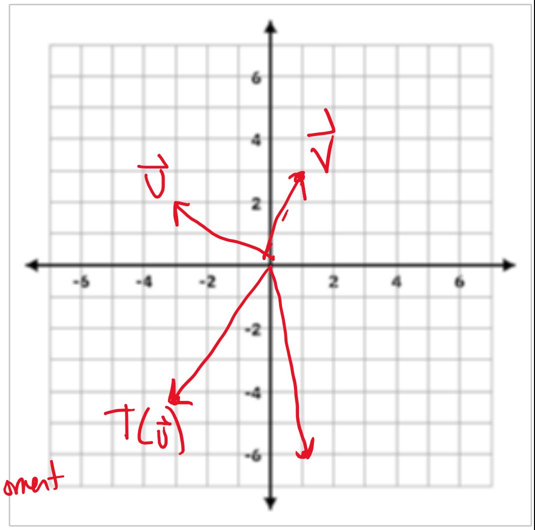
CALCULATOR ACTIVE:Directions: Function *f* is the given linear transformation. Identify the matrix expression that would determine the result of the given transformation.

3) T $(\langle x, y \rangle) \to \langle x, 2x + y \rangle$. Identify the matrix expression that would determine the result of T: $\langle 0,5 \rangle$	4) T $(\langle x, y \rangle) \to \langle 2x - 3y, 3x \rangle$. Identify the matrix expression that would determine the result of T: $\langle 2,-3 \rangle$

CA #1 SOLUTIONS

Directions: Apply the transformation for each matrix A.

1) Apply the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ if $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$.



a. If $\vec{u} = \langle -3,2 \rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix} \quad \langle -3,-4 \rangle$$

b. If $\vec{v} = \langle 1,3 \rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix} \quad \langle 1,-6 \rangle$$

c. Describe the transformation that occurs.

Reflection in x-axis and dilation of y component

d. What is the general transformation that occurs to $\langle x, y \rangle$.

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -2y \end{bmatrix} = \langle x,-2y \rangle$$

Directions: Determine if T: $\mathbb{R}^2 \to \mathbb{R}^2$ given by T $(\langle x, y \rangle)$ is a linear transformation.

2) **T $(\langle x, y \rangle) \to \langle 2x, x-y \rangle$**

$\vec{u} = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

$$T(\vec{u}+\vec{v}) \stackrel{?}{=} T(\vec{u}) + T(\vec{v})$$

$$T\begin{bmatrix} a_1+a_2 \\ b_1+b_2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2a_1 \\ a_1-b_1 \end{bmatrix} + \begin{bmatrix} 2a_2 \\ a_2-b_2 \end{bmatrix}$$

$$\begin{bmatrix} 2a_1+2a_2 \\ (a_1+a_2)-(b_1+b_2) \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2a_1+2a_2 \\ a_1-b_1+a_2-b_2 \end{bmatrix}$$

$$\begin{bmatrix} 2a_1+2a_2 \\ a_1-b_1+a_2-b_2 \end{bmatrix} = \checkmark$$

$$cT(\vec{u}) \stackrel{?}{=} T(c\vec{u})$$

$$c\begin{bmatrix} 2a_1 \\ a_1-b_1 \end{bmatrix} = T\left(\begin{bmatrix} ca_1 \\ cb_1 \end{bmatrix}\right)$$

$$\begin{bmatrix} c2a_1 \\ ca_1-cb_1 \end{bmatrix} = \begin{bmatrix} 2ca_1 \\ ca_1-cb_1 \end{bmatrix} \checkmark$$

CALCULATOR ACTIVE: Directions: Function *f* is the given linear transformation. Identify the matrix expression that would determine the result of the given transformation.

3) T $(\langle x, y \rangle) \to \langle x, 2x+y \rangle$.

Identify the matrix expression that would determine the result of T: $\langle 0,5 \rangle$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 5 \end{bmatrix}}$$

4) T $(\langle x, y \rangle) \to \langle 2x-3y, 3x \rangle$.

Identify the matrix expression that would determine the result of T: $\langle 2,-3 \rangle$

$$\begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -\frac{4}{3} \end{bmatrix}$$