4.12 Linear Transformations and Matrices

AP Precalculus   Name: ______________________________

CA #2

Directions: Apply the transformation for each matrix *A*.

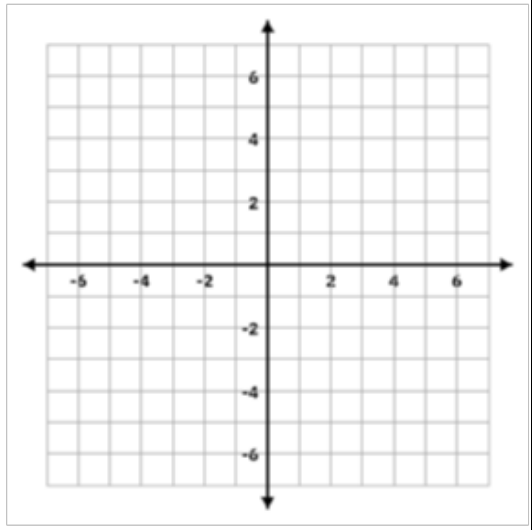
1) Apply the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ if $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

   a. If $\vec{u} = \langle 5, 3\rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

   b. If $\vec{v} = \langle -4, \ -5\rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

   c. Describe the transformation that occurs.

   d. What is the general transformation that occurs to $\langle x, y\rangle$.



Directions: Determine if T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by T $(\langle x, y\rangle)$ is a linear transformation.

2) **T** $(\langle \boldsymbol{x}, \boldsymbol{y}\rangle) \rightarrow \langle \boldsymbol{y}^2, 2\boldsymbol{x}\rangle$

Directions: Function *f* is the given linear transformation. Identify the matrix expression that would determine the result of the given transformation.

3) T $(\langle x, y\rangle) \rightarrow \langle x - y, x + y\rangle$.

Identify the matrix expression that would determine the result of T: $\langle 3, -2\rangle$

4) T $(\langle x, y\rangle) \rightarrow \langle 2x + y, 3x - 3y\rangle$.

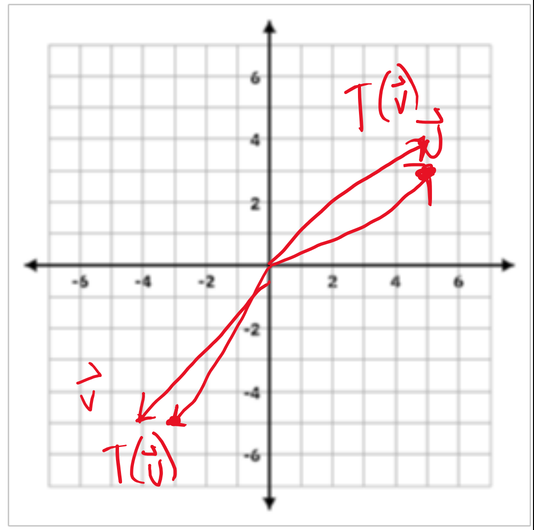
Identify the matrix expression that would determine the result of T: $\langle -5, 6\rangle$

Directions: Apply the transformation for each matrix A.

1) Apply the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ if $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

a. If $\vec{u} = \langle 5, 3 \rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$ $\langle -3, -5 \rangle$



b. If $\vec{v} = \langle -4, -5 \rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -4 \\ -5 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $\langle 5, 4 \rangle$

c. Describe the transformation that occurs.

Reflection in $y = -x$

d. What is the general transformation that occurs to $\langle x, y \rangle$.

$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$ $\langle -y, -x \rangle$

$\vec{u} = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$

Directions: Determine if T: $\mathbb{R}^2 \to \mathbb{R}^2$ given by T $(\langle x, y \rangle)$ is a linear transformation.

$\vec{v} = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

$\vec{u}+\vec{v} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \end{bmatrix}$

2) $\mathbf{T}(\langle x, y \rangle) \to \langle y^2, 2x \rangle$

$T(\vec{u}+\vec{v}) \overset{?}{=} T(\vec{u}) + T(\vec{v})$

$T\begin{bmatrix} a_1+a_2 \\ b_1+b_2 \end{bmatrix} \overset{?}{=} \begin{bmatrix} b_1^2 \\ 2a_1 \end{bmatrix} + \begin{bmatrix} b_2^2 \\ 2a_2 \end{bmatrix}$

$\begin{bmatrix} (b_1+b_2)^2 \\ 2(a_1+a_2) \end{bmatrix} = \begin{bmatrix} b_1^2+b_2^2 \\ 2a_1+2a_2 \end{bmatrix}$

$\begin{bmatrix} b_1^2+2b_1b_2+b_2^2 \\ 2a_1+2a_2 \end{bmatrix} =$ ✗

NOT A TRANSFORMATION

Directions: Function *f* is the given linear transformation. Identify the matrix expression that would determine the result of the given transformation.

3) T $(\langle x, y \rangle) \to \langle x - y, x + y \rangle$.

Identify the matrix expression that would determine the result of T: $\langle 3, -2 \rangle$

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$= \begin{bmatrix} 1/2 \\ -5/2 \end{bmatrix}$

4) T $(\langle x, y \rangle) \to \langle 2x + y, 3x - 3y \rangle$.

Identify the matrix expression that would determine the result of T: $\langle -5, 6 \rangle$

$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}^{-1} \times \begin{bmatrix} -5 \\ 6 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$= \begin{bmatrix} -1 \\ -3 \end{bmatrix}$