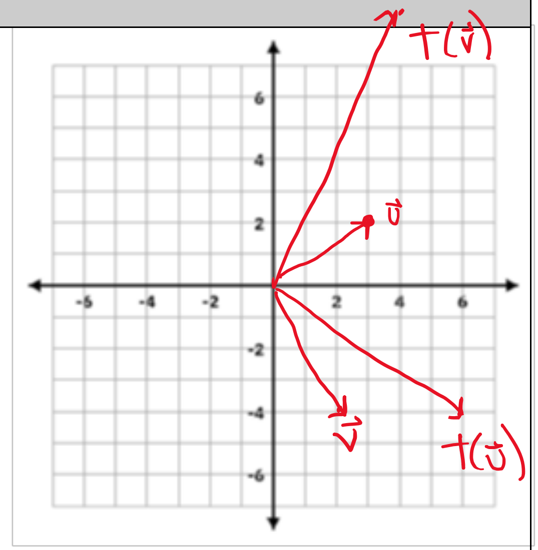
4.12 Linear Transformations and Matrices

AP Precalculus

4.12 Practice Solutions

Directions: Apply the transformation for each matrix *A*.

1) Apply the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ if $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$.



a. If $\vec{u} = \langle 3, 2 \rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

b. If $\vec{v} = \langle 2, -4 \rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

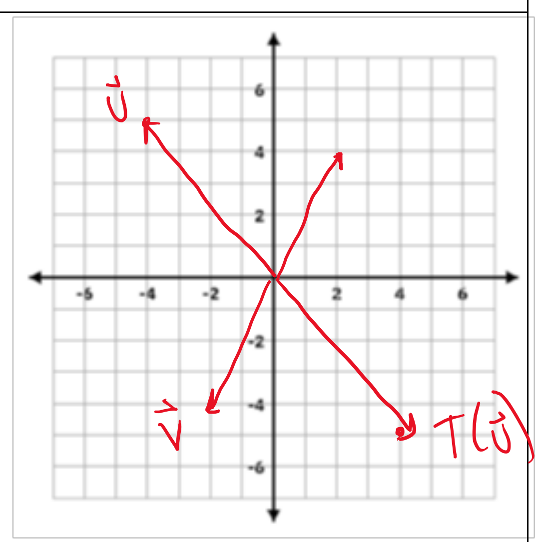
$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

c. Describe the transformation that occurs. Reflection across x-axis and vertical scalar

d. What is the general transformation that occurs to $\langle x, y \rangle$.

$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ -2y \end{bmatrix}$$

2) Apply the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ if $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.



a. If $\vec{u} = \langle -4, 5 \rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

b. If $\vec{v} = \langle -2, -4 \rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

c. Describe the transformation that occurs.

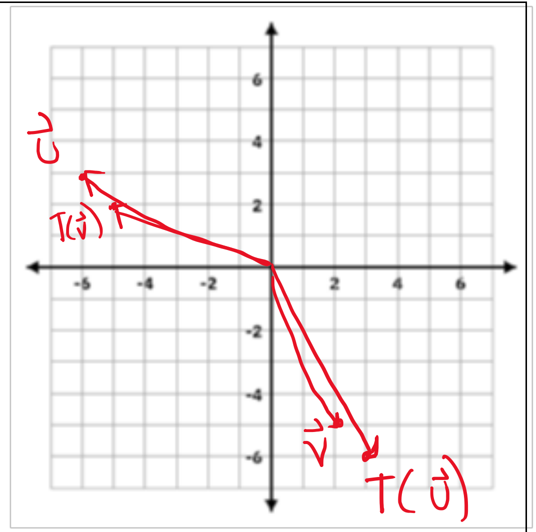
Reflect in x and y-axis

d. What is the general transformation that occurs to $\langle x, y \rangle$. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$

3) Apply the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ if $A = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$.

a. If $\vec{u} = \langle -6, 3\rangle$, find $T(\vec{u})$ and sketch both $\vec{u}$ and $T(\vec{u})$.

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}-6\\ 3\end{bmatrix} = \begin{bmatrix}3\\ -6\end{bmatrix}$$



b. If $\vec{v} = \langle 2, -5\rangle$, find $T(\vec{v})$ and sketch both $\vec{v}$ and $T(\vec{v})$.

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}2\\ -5\end{bmatrix} = \begin{bmatrix}-5\\ 2\end{bmatrix}$$

c. Describe the transformation that occurs.

Reflection in $y = x$.

d. What is the general transformation that occurs to $\langle x, y\rangle$.

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}y\\ x\end{bmatrix}$$

Directions: Determine if T: $\mathbb{R}^2 \to \mathbb{R}^2$ given by T $(\langle x, y\rangle)$ is a linear transformation.

$\vec{u} = \begin{bmatrix}a_1\\ b_1\end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix}a_2\\ b_2\end{bmatrix}$

4) **T $(\langle x, y\rangle) \to \langle xy, x + y\rangle$**

$$T(\vec{u} + \vec{v}) \stackrel{?}{=} T(\vec{u}) + T(\vec{v})$$

$$T\left(\begin{bmatrix}a_1 + a_2\\ b_1 + b_2\end{bmatrix}\right) = \begin{bmatrix}a_1b_1\\ a_1 + b_1\end{bmatrix} + \begin{bmatrix}a_2b_2\\ a_2 + b_2\end{bmatrix}$$

$$\begin{bmatrix}(a_1 + a_2)(b_1 + b_2)\\ a_1 + a_2 + b_1 + b_2\end{bmatrix} = \begin{bmatrix}a_1b_1 + a_2b_2\\ a_1 + b_1 + a_2 + b_2\end{bmatrix}$$

$$\begin{bmatrix}a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2\\ a_1 + a_2 + b_1 + b_2\end{bmatrix} = \times$$

5) **T $(\langle x, y\rangle) \to \langle -y, -x\rangle$**

$$T\left(\begin{bmatrix}a_1 + a_2\\ b_1 + b_2\end{bmatrix}\right) \stackrel{?}{=} \begin{bmatrix}-b_1\\ -a_1\end{bmatrix} + \begin{bmatrix}-b_2\\ -a_2\end{bmatrix}$$

$$\begin{bmatrix}-(b_1 + b_2)\\ -(a_1 + a_2)\end{bmatrix} = \begin{bmatrix}-b_1 + -b_2\\ -a_1 + -a_2\end{bmatrix}$$

$$\begin{bmatrix}-b_1 + -b_2\\ -a_1 + -a_2\end{bmatrix} = \checkmark$$

$$cT(\vec{u}) = T(c\vec{u})$$

$$c\begin{bmatrix}-b_1\\ -a_1\end{bmatrix} = T\begin{bmatrix}ca_1\\ cb_1\end{bmatrix}$$

$$\begin{bmatrix}-cb_1\\ -ca_1\end{bmatrix} = \begin{bmatrix}-cb_1\\ -ca_1\end{bmatrix} \checkmark$$

Directions: Function *f* is the given linear transformation. Identify the matrix expression that would determine the result of the given transformation.	
7) T $(\langle x, y\rangle) \rightarrow \langle x - y, x + 3y\rangle$. Identify the matrix expression that would determine the result of T: $\langle 1,-4\rangle$ $\begin{bmatrix}1&-1\\1&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\-4\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1&-1\\1&3\end{bmatrix}^{-1}\times\begin{bmatrix}1\\-4\end{bmatrix}$ $\boxed{\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-\frac{1}{4}\\-\frac{5}{4}\end{bmatrix}}$	8) T $(\langle x, y\rangle) \rightarrow \langle x - 2y, -2x\rangle$. Identify the matrix expression that would determine the result of T: $\langle 0, 6\rangle$ $\begin{bmatrix}1&-2\\-2&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\6\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1&-2\\-2&0\end{bmatrix}^{-1}\times\begin{bmatrix}6\\6\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-3\\-\frac{3}{2}\end{bmatrix}$
9) T $(\langle x, y\rangle) \rightarrow \langle -y, x\rangle$. Identify the matrix expression that would determine the result of T: $\langle 10,-3\rangle$ $\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}10\\-3\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}^{-1}\times\begin{bmatrix}10\\-3\end{bmatrix}$ $\boxed{\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-3\\10\end{bmatrix}}$	10) T $(\langle x, y\rangle) \rightarrow \langle 2x + 3y, 2x-y\rangle$. Identify the matrix expression that would determine the result of T: $\langle 1,-1\rangle$ $\begin{bmatrix}2&3\\2&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2&3\\2&-1\end{bmatrix}^{-1}\times\begin{bmatrix}1\\-1\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-\frac{1}{4}\\\frac{1}{2}\end{bmatrix}$

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4.12 Test Prep

11. (2.13A) Solve the equation $\log_b a + \log_b 5 = c$ for a.

$\log_b 5a = c$

$\frac{b^c}{5} = \frac{5a}{5}$

(A) $\frac{5}{b^c}$

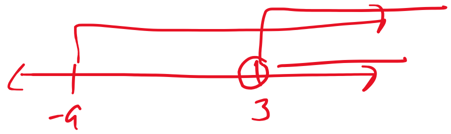
(B) $5b^c$

(C) $b^c - 5$

(D) $\frac{b^c}{5}$

$x > 3$ $\quad$ $x > -9$

12. (2.13B) When considering the equation $\log(x-3) + \log(5) > \log(x+9)$, which of the following represents the domain of all solutions to the inequality?



(A) $(3, \infty)$

(B) $(5, \infty)$

(C) $(-9, \infty)$

(D) $(6, \infty)$

13. (2.10) Which of the following represents a possible function that is the inverse of $f(x) = 0.25^x$?

(A)

x	$f(x)$
-3	64
-2	16
-1	4
0	1

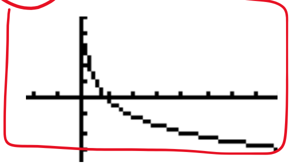
$f(x) = .25^x$

(B) $f^{-1}(x)$

x	$f(x)$
4	-1
1	0
$\frac{1}{4}$	1
$\frac{1}{16}$	2

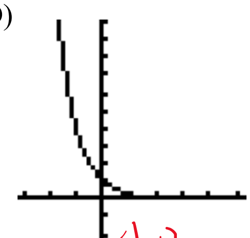
multiplication is on the x-values

(C) $f^{-1}(x)$



x-values increase multiplicatively

(D)



This is $f(x) = .25^x$