4.13A Matrices as Functions

AP Precalculus

4.13A Practice Solutions

Directions: Given the linear transformation, find the associated matrix with that transformation.

1) <x, y> to <x - y, x + y>

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

2) <x, y> to <2x + 3y, 3x + 9y>

$$\begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix}$$

3) <x, y> to <-y, 4x + 5y>

$$\begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix}$$

Directions: Find the linear transformation given the associated matrix.

4) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\langle x, y\rangle$ to $\langle 2x, 2y\rangle$

5) $\begin{bmatrix} 3 & 4 \\ 1 & -3 \end{bmatrix}$

$\langle x, y\rangle$ to $\langle 3x+4y, x-3y\rangle$

6) $\begin{bmatrix} 5 & 0 \\ -3 & -2 \end{bmatrix}$

$\langle x, y\rangle$ to $\langle 5x, -3x-2y\rangle$

Directions: Find the resulting vector from rotating the given vector by the given angle.

7) $\vec{u} = \langle 2, -3\rangle$ rotated $\frac{\pi}{2}$ radians counterclockwise.

$$\begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\boxed{\langle 3, 2\rangle}$$

8) $\vec{v} = \langle -4, 8\rangle$ rotated $\frac{\pi}{4}$ radians counterclockwise.

$$\begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}\begin{bmatrix} -4 \\ 8 \end{bmatrix} = \begin{bmatrix} -\frac{4\sqrt{2}}{2} - \frac{8\sqrt{2}}{2} \\ -\frac{4\sqrt{2}}{2} + \frac{8\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} -\frac{12\sqrt{2}}{2} \\ \frac{4\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} -6\sqrt{2} \\ 2\sqrt{2} \end{bmatrix}$$

$$\langle -6\sqrt{2}, 2\sqrt{2}\rangle$$

9) $\vec{v} = \langle 4, 1\rangle$ rotated $\frac{\pi}{6}$ radians counterclockwise.

$$\begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{4\sqrt{3}}{2} - \frac{1}{2} \\ 2 + \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\left\langle \frac{4\sqrt{3}-1}{2}, 2 + \frac{\sqrt{3}}{2} \right\rangle$$

10) $\vec{u} = \langle -6, -4\rangle$ rotated π radians counterclockwise.

$$\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -6 \\ -4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$\langle 6, 4\rangle$$

Directions: Considering the given transformation, what is the image of the given vector under the transformation.	
11) The x- and y-coordinates are dilated by a factor of 4 and $\vec{u} = \langle 3,-2\rangle$ $\langle 4x, 4y\rangle$ $\begin{bmatrix}4&0\\0&4\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix}=\begin{bmatrix}12\\-8\end{bmatrix}$ $\langle 12,-8\rangle$	12) $\langle x,y\rangle$ to $\langle x+2y,\ -2x-y\rangle$ *and* $\vec{u} = \langle 4,6\rangle$ $\begin{bmatrix}1&2\\-2&-1\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix}=\begin{bmatrix}4+12\\-8-6\end{bmatrix}=\begin{bmatrix}16\\-14\end{bmatrix}$ $\langle 16,-14\rangle$
13) The x-coordinate doubles, the y-coordinate quadruples and $\vec{u} = \langle -1,-5\rangle$ $\langle 2x, 4y\rangle$ $\begin{bmatrix}2&0\\0&4\end{bmatrix}\begin{bmatrix}-1\\-5\end{bmatrix}=\begin{bmatrix}-2\\-20\end{bmatrix}$ $\langle -2,-20\rangle$	14) $\langle x,y\rangle$ to $\langle 3x-y,\ -x+7y\rangle$ *and* $\vec{u} = \langle -3,3\rangle$ $\begin{bmatrix}3&-1\\-1&7\end{bmatrix}\begin{bmatrix}-3\\3\end{bmatrix}=\begin{bmatrix}-9+-3\\3+21\end{bmatrix}=\begin{bmatrix}-12\\24\end{bmatrix}$ $\langle -12,24\rangle$

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4.13A Test Prep

15. (3.2B) An angle is ~~in standard~~ position in the xy-plane. Which of the following is true about θ on the interval 0≤θ≤2π if cos θ <0?

So cosθ is negative where does that happen?

cos ⇒ x-values

(A) There is no value θ of on 0≤θ≤2π for which cos <0.

(B) There are values θ of on 0≤θ≤2π for which cos θ <0 in all four Quadrants.

(C) There is a value of θ on 0≤θ≤2π for which cos θ <0 in Quadrant II only.

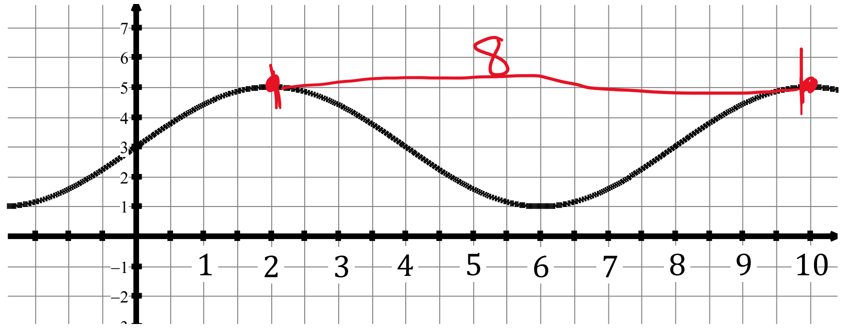
(D) There are values of θ on 0≤θ≤2π for which cos θ <0 in Quadrants II and III only.

16. (3.5) The figure shows the graph of a periodic function f in the xy-plane. What is the frequency of f?

FREQ $= \frac{1}{\text{period}}$

per $= 8$

so $1/8$



Graph of f

(A) $\frac{1}{8}$ (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{4}$ (D) 8

17. (3.6A) The table gives ordered pairs for seven points from a larger data set. The larger data set can be modeled by a sinusoidal function f with a period of 6. The minimum values of the data set occur at x-values that are multiples of 6.

x	0	1	2	3	4	5	6
$f(x)$	-4	-1	3	6	3	-1	-4

Which of the following best defines $f(x)$ for the larger data set?

(A) ~~$-4\cos(12\pi x) + 1$~~

(B) $-4\cos\left(\frac{\pi}{3}x\right) + 1$

(C) ~~$-5\cos(12\pi x) + 1$~~

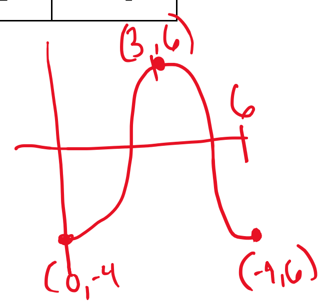
(D) $-5\cos\left(\frac{\pi}{3}x\right) + 1$

Per $= 6$

$6 = 2\pi/b$

$b = 2\pi/6$

$b = \pi/3$



amp $= \frac{6 - -4}{2}$

$= \frac{10}{2}$

$= 5$