AP Precalc

## Composition of Two Linear Transformations

You apply one transformation and then the other one.
Transformation $T$ and $U$ denoted by $T(U(\vec{v}))$ means we apply $U$ then $T$.

If Transformation $T$ is associated with matrix $A$, and transformation $U$ is associated with matrix $B$, then the product of the matrices is the composition of the overall transformation.

$$
T(U(\vec{v}))=A B
$$

Ex 1: Matrix A and B represent the transformations T and U respectively. Find the associated matrix for the composition of the function and then find $T(U(\vec{v}))$
$A=\left[\begin{array}{ll}3 & 2 \\ 0 & 1\end{array}\right], B=\left[\begin{array}{cc}4 & 6 \\ -2 & 3\end{array}\right], \vec{v}=\langle 4,3\rangle$

Ex 2: Find the associated matrix for the composition of the function and then find $U(T(\vec{v}))$.
$A=\left[\begin{array}{ll}3 & 2 \\ 0 & 1\end{array}\right], B=\left[\begin{array}{cc}4 & 6 \\ -2 & 3\end{array}\right], \vec{v}=\langle 4,3\rangle$

Ex 3: Find the associated matrix to the composition of transformations with reflection across the x -axis and a rotation of $\frac{\pi}{2}$.

Suppose $T$ is a linear transformation represented by the matrix $A=\left[\begin{array}{ll}1 & 1 \\ 4 & 2\end{array}\right]$. Given $\vec{v}=\langle 10,4\rangle$ find the vector, $\vec{u}$ that was transformed by $A$ to get $\vec{v}$.

### 4.13B Matrices as Functions

AP Precalculus

### 4.13B Practice

Directions: Matrix A and B represent the transformations $T$ and $U$ respectively. Find the associated matrix for the composition of the function and then find the vector after the given transformation.

1) Find the associated matrix and $T(U(\stackrel{\rightharpoonup}{v}))$.
$A=\left[\begin{array}{cc}-2 & -1 \\ 3 & 5\end{array}\right], B=\left[\begin{array}{cc}0 & 2 \\ -4 & 1\end{array}\right], \vec{v}=\langle 2,-5\rangle$
2) Find the associated matrix and $T(U(\vec{v}))$.
$A=\left[\begin{array}{cc}4 & -2 \\ -4 & 3\end{array}\right], B=\left[\begin{array}{ll}1 & 3 \\ 3 & 1\end{array}\right], \vec{v}=\langle-3,-4\rangle$
3) Find the associated matrix and $U(T(\stackrel{\rightharpoonup}{v}))$.
$A=\left[\begin{array}{cc}-2 & -1 \\ 3 & 5\end{array}\right], B=\left[\begin{array}{cc}0 & 2 \\ -4 & 1\end{array}\right], \vec{v}=\langle 2,-5\rangle$
4) Find the associated matrix and $U(T(\vec{v}))$.
$A=\left[\begin{array}{cc}4 & -2 \\ -4 & 3\end{array}\right], B=\left[\begin{array}{ll}1 & 3 \\ 3 & 1\end{array}\right], \vec{v}=\langle-3,-4\rangle$

| 5) Reflect across the y-axis and a rotation of | 6) Rotation of $\frac{\pi}{2}$ radians counterclockwise and a <br> horizontal and vertical dilation of 4 units. |
| :--- | :--- |
| radians counterclockwise. |  |

Directions: Given $\overrightarrow{\boldsymbol{v}}$ find the vector $\overrightarrow{\boldsymbol{u}}$, that was transformed by matrix $\boldsymbol{A}$ to get $\overrightarrow{\boldsymbol{v}}$.
7) $\vec{v}=\langle-4,2\rangle$ and $A=\left[\begin{array}{ll}3 & 2 \\ 2 & 2\end{array}\right]$.
8) $\vec{v}=\langle 9,-3\rangle$ and $A=\left[\begin{array}{ll}6 & -5 \\ 3 & -3\end{array}\right]$.
9) $\vec{v}=\langle-2,-3\rangle$ and $A=\left[\begin{array}{ll}-4 & -3 \\ -3 & -2\end{array}\right]$.
10) (3.8) The graph of $f(x)=\tan (b x)$, where $b$ is a constant, is shown in the $x y$-plane. What is the value of $b$ ?
(A) 4
(B) 2
(C) $\frac{\pi}{2}$
(D) $\frac{\pi}{4}$

11) (3.10) The function $g$ is given by $g(x)=2 \cos (x)$. What are all solutions $g(x)=\sqrt{3}$ ?
(A) $x=\frac{\pi}{6}+2 \pi k$ and $\frac{5 \pi}{6}+2 \pi k$, where $k$ is any integer
(B) $x= \pm \frac{\pi}{6}+2 \pi k$, where $k$ is any integer
(C) $x=\frac{\pi}{3}+2 \pi k$ and $\frac{2 \pi}{3}+2 \pi k$, where $k$ is any integer
(D) $x= \pm \frac{\pi}{3}+2 \pi k$, where $k$ is any integer
12) (3.13) The point $A$ has polar coordinates $\left(4, \frac{7 \pi}{6}\right)$. Which of the following also gives the location of point $A$ in polar coordinates?
(A) $\left(4,-\frac{11 \pi}{6}\right)$
(B) $\left(4,-\frac{5 \pi}{6}\right)$
(C) $\left(-4,-\frac{\pi}{6}\right)$
(D) $\left(-4,-\frac{5 \pi}{6}\right)$

