4.13B Matrices as Functions

AP Precalculus

4.13B Practice Solutions

Directions: Matrix A and B represent the transformations T and U respectively. Find the associated matrix for the composition of the function and then find the vector after the given transformation.

1) Find the associated matrix and $T(U(\vec{v}))$.

$A = \begin{bmatrix} -2 & -1 \\ 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix}, \vec{v} = \langle 2, -5 \rangle$

$AB = \begin{bmatrix} -2 & -1 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -20 & 11 \end{bmatrix}$

$T(U(\vec{v})) = \begin{bmatrix} 4 & -5 \\ -20 & 11 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 33 \\ -95 \end{bmatrix}$

2) Find the associated matrix and $U(T(\vec{v}))$.

$A = \begin{bmatrix} -2 & -1 \\ 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix}, \vec{v} = \langle 2, -5 \rangle$

$BA = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 11 & 9 \end{bmatrix}$

$U(T(\vec{v})) = \begin{bmatrix} 6 & 10 \\ 11 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} -38 \\ -23 \end{bmatrix}$

3) Find the associated matrix and $T(U(\vec{v}))$.

$A = \begin{bmatrix} 4 & -2 \\ -4 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}, \vec{v} = \langle -3, -4 \rangle$

$AB = \begin{bmatrix} 4 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ 5 & -9 \end{bmatrix}$

$T(U(\vec{v})) = \begin{bmatrix} -2 & 10 \\ 5 & -9 \end{bmatrix}\begin{bmatrix} -3 \\ -4 \end{bmatrix} = \begin{bmatrix} -34 \\ 21 \end{bmatrix}$

4) Find the associated matrix and $U(T(\vec{v}))$.

$A = \begin{bmatrix} 4 & -2 \\ -4 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}, \vec{v} = \langle -3, -4 \rangle$

$BA = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -8 & 7 \\ 8 & -3 \end{bmatrix}$

$U(T(\vec{v})) = \begin{bmatrix} -8 & 7 \\ 8 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ -4 \end{bmatrix} = \begin{bmatrix} -4 \\ -12 \end{bmatrix}$

Directions: Find the associated matrix to the composition of transformations.

5) Reflect across the y-axis and a rotation of π radians counterclockwise.

$A = \langle -x, y \rangle = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$

$B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$BA = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

6) Rotation of $\frac{\pi}{2}$ radians counterclockwise and a horizontal and vertical dilation of 4 units.

$\langle 4x + 0y, 0x + 4y \rangle$

$\langle 4x, 4y \rangle$

$A = \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$

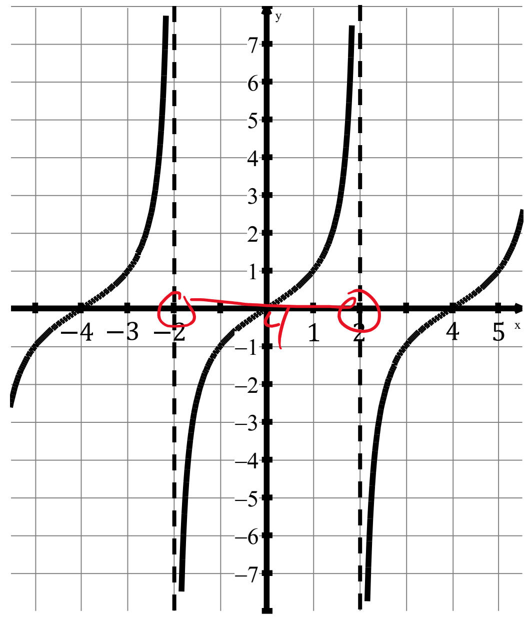
$B = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$

Directions: Given $\vec{v}$ find the vector $\vec{u}$, that was transformed by matrix A to get $\vec{v}$.
7) $\vec{v} = \langle -4,2 \rangle$ and $A = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$. $A \times \vec{u} = \vec{v}$ $\vec{u} = A^{-1} \times \vec{v}$ $A^{-1} = \frac{1}{6-4}\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$ $= \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & \frac{3}{2} \end{bmatrix}$ $\vec{u} = \begin{bmatrix} 1 & -1 \\ -1 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} -6 \\ 7 \end{bmatrix}$ $\vec{u} = \langle -6, 7 \rangle$
8) $\vec{v} = \langle 9,-3 \rangle$ and $A = \begin{bmatrix} 6 & -5 \\ 3 & -3 \end{bmatrix}$. $A \times \vec{u} = \vec{v}$ $\vec{u} = A^{-1} \times \vec{v}$ $A^{-1} = \frac{1}{-18+15}\begin{bmatrix} -3 & 5 \\ -3 & 6 \end{bmatrix}$ $= \frac{1}{-3}\begin{bmatrix} -3 & 5 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{5}{3} \\ 1 & -2 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} 1 & -\frac{5}{3} \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ -3 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} 14 \\ 15 \end{bmatrix}$ $\vec{u} = \langle 14, 15 \rangle$
9) $\vec{v} = \langle -2,-3 \rangle$ and $A = \begin{bmatrix} -4 & -3 \\ -3 & -2 \end{bmatrix}$. $A \times \vec{u} = \vec{v}$ $\vec{u} = A^{-1} \times \vec{v}$ $A^{-1} = \frac{1}{8-9}\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ $= -1\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ $= \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$ $\vec{u} = \langle 5, -6 \rangle$

10) (3.8) The graph of $f(x) = \tan(bx)$, where b is a constant, is shown in the xy-plane. What is the value of b?



Graph of f

Per = 4

$\frac{\pi}{b} = 4$

$\frac{\pi}{4} = b$

(A) 4

(B) 2

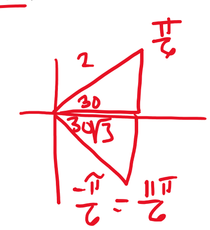
(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

11) (3.10) The function g is given by $g(x) = 2\cos(x)$. What are all solutions $g(x) = \sqrt{3}$?

$\sqrt{3} = 2\cos x$

adj/hyp $\frac{\sqrt{3}}{2} = \cos x$



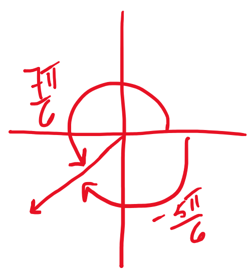
(A) $x = \frac{\pi}{6} + 2\pi k$ and $\frac{5\pi}{6} + 2\pi k$, where k is any integer

(B) $x = \pm\frac{\pi}{6} + 2\pi k$, where k is any integer

(C) $x = \frac{\pi}{3} + 2\pi k$ and $\frac{2\pi}{3} + 2\pi k$, where k is any integer

(D) $x = \pm\frac{\pi}{3} + 2\pi k$, where k is any integer

12) (3.13) The point A has polar coordinates $\left(4, \frac{7\pi}{6}\right)$. Which of the following also gives the location of point A in polar coordinates?



(A) $\left(4, -\frac{11\pi}{6}\right)$

(B) $\left(4, -\frac{5\pi}{6}\right)$

(C) $\left(-4, -\frac{\pi}{6}\right)$

(D) $\left(-4, -\frac{5\pi}{6}\right)$