4.14 Matrices Modeling Contexts

AP Precalculus

4.14 Practice Solutions

CALCULATOR ACTIVE: Directions: Use the given information to answer the questions.

1) AirBnB did some market research and found that in the southeast United States, they roughly lost about 10% of its customers to hotels on their next stay. The hotels confirmed that they lost about 24% of its customers to AirBnB on their customers' next stay. Currently hotels account for about 70% of all stays in the given area, while AirBnB accounts for about 20%.

A) Find a transition matrix.

$$\begin{array}{c} \\ A \\ H \end{array}\begin{bmatrix} A & H \\ .90 & .24 \\ .10 & .76 \end{bmatrix}$$

B) What percent of all night stays does each company account for after 1 year? 3 years?

$$\begin{bmatrix} .90 & .24 \\ .10 & .76 \end{bmatrix}\begin{bmatrix} .20 \\ .70 \end{bmatrix} = \begin{bmatrix} .348 \\ .552 \end{bmatrix}$$ 1 year $\langle .348, .552 \rangle$ AirBnB

$$\begin{bmatrix} .90 & .24 \\ .10 & .76 \end{bmatrix}^2\begin{bmatrix} .20 \\ .70 \end{bmatrix} = \begin{bmatrix} .446 \\ .454 \end{bmatrix}$$ 2 years $\langle .446, .454 \rangle$ A H

C) What will be the eventual long-term distribution of night stays for the companies?

$\langle .635, .2657 \rangle$ A H

D) What percent did each company account for last year?

$$\begin{bmatrix} .9 & .24 \\ .1 & .76 \end{bmatrix}^{-1} \times \begin{bmatrix} .20 \\ .70 \end{bmatrix} = \begin{bmatrix} -0.02 \\ .92 \end{bmatrix}$$ $\langle -.02, .92 \rangle$ A H

Not possible!

2) Mr. Sullivan and Mr. Kelly are the only Algebra 2 teachers at their school. Mr. Kelly discovers that from one semester to the next 7% of his students move from his class to Mr. Sullivan's, while 5% of Mr. Sullivan's students switch to Mr. Kelly's at the same time. This year Mr. Kelly had 60% of all students in Algebra 2.

A) Find a transition matrix.

$$\begin{array}{c} \\ K \\ S \end{array}\begin{bmatrix} K & S \\ .93 & .05 \\ .07 & .95 \end{bmatrix}$$

B) What percent of all Algebra 2 students will be in their classes next year?

$$\begin{bmatrix} .93 & .05 \\ .07 & .95 \end{bmatrix}\begin{bmatrix} .6 \\ .4 \end{bmatrix} = \begin{bmatrix} .578 \\ .422 \end{bmatrix}$$ $\langle .578, .422 \rangle$ K S

C) What percent of all Algebra 2 students were in their classes last year?

$$\begin{bmatrix} .93 & .05 \\ .07 & .95 \end{bmatrix}^{-1} \times \begin{bmatrix} .6 \\ .4 \end{bmatrix} = \begin{bmatrix} .625 \\ .375 \end{bmatrix}$$ $\langle .625, .375 \rangle$ K S

D) Is there a steady state for the percent of students in each teachers' classes? What is it?

I compared between A^{50} and A^{60}

$\langle .417, .583 \rangle$ K S

3) Lebron James is the GOAT! When he makes a shot there is a 62% chance that he makes the next one. If he misses a shot there is a 44% chance that he'll make the next one. He makes about 54% of all of his shots.

A) Find a transition matrix.

	Make	Miss
Make	.62	.44
Miss	.38	.56

B) What are the probabilities that he will make/miss his next shot?

$$\begin{bmatrix} .62 & .44 \\ .38 & .56 \end{bmatrix}\begin{bmatrix} .54 \\ .46 \end{bmatrix} = \begin{bmatrix} .537 \\ .463 \end{bmatrix} \quad \langle .537, .463 \rangle \text{ Make, Miss}$$

C) What is the long term distribution for Lebron's shot making for his next shot?

$\langle .537, .463 \rangle$ Make, Miss

4) Mr. Brust has been studying the migration patterns of the American Beaver. He concludes that from one season to the next about 25% of the American Beaver population will migrate from the Wetlands to the Riverlands, with the rest staying. As well, about 10% of the American Beavers will migrate from the Riverlands to the Wetlands, with the rest staying. In 2023, Mr. Brust found 1200 American Beavers living in the Wetlands and 3500 living in the Riverlands.

A) Is there a steady state of the American Beavers in these ecosystems? If so, what is it?

	W	R
W	.75	.10
R	.25	.90

B) What was the population distribution of the American Beavers last year?

$$\begin{bmatrix} .75 & .10 \\ .25 & .90 \end{bmatrix}^{-1}\begin{bmatrix} 1200 \\ 3500 \end{bmatrix} = \begin{bmatrix} 1123 \\ 3577 \end{bmatrix} \quad \langle 1123, 3577 \rangle \text{ W, R}$$

C) What will be the population distribution in 2026?

2023 3500

$$\begin{bmatrix} .75 & .10 \\ .25 & .90 \end{bmatrix}^{3}\times\begin{bmatrix} 1200 \\ 3500 \end{bmatrix} = \begin{bmatrix} 1304 \\ 3396 \end{bmatrix} \quad \langle 1304, 3396 \rangle \text{ W, R}$$

5) The Evil Empire has had a tough time with retention of its stormtroopers. This year 22% of its stormtrooper recruits left to join the Rebellion with the remaining ones scared to leave. The same year the Evil Empire was able to use the dark side of the force to sway 8% of the Rebellion's recruits to join them as stormtroopers while the rest of the Rebellion stayed strong to help Luke, Han and Chewie. The Dark Side had 30,000 recruits and the Rebellion had 2500 recruits this year.

A) How many recruits will each have in 2 years from this group?

	EE	R
EE	.78	.08
R	.22	.92

B) How many recruits did each group have last year?

$$\begin{bmatrix} .78 & .08 \\ .22 & .92 \end{bmatrix}^{-1}\begin{bmatrix} 30{,}000 \\ 2500 \end{bmatrix} = \begin{bmatrix} 39143 \\ -6643 \end{bmatrix}$$ DOESN'T WORK

C) Is there a steady state in this situation? If so, what is it?

$\langle 8667, 23833 \rangle$ EE, R

4.14 Matrices as Functions

6) (1.9) The function f is a rational function graphed in the xy-plane. The polynomial in the numerator of f has exactly one real zero at $x = 3$. The polynomial of the denominator of f has exactly two real zeros at both $x = 3$ and $x = 6$. The multiplicities of the zeros at $x = 3$ in the numerator and in the denominator are equal.

a. Find the domain for the graph of f.

$(-\infty,3)\cup(3,6)\cup(6,\infty)$

b. Describe any holes and/or vertical asymptotes for the graph of f.

x=3 is a hole because the polynomial in the numerator and the polynomial in the denominator share a common factor (x-3).

x=6 is a vertical asymptote because of the zero in the denominator.

c. Explain how your answer from part b would change if the multiplicities of the zeros at $x = 3$ in the numerator and denominator were not equal?

If the multiplicity of the zero in the numerator was greater than the denominator, then there would still be a hole at x=3. The overall graph might look different, but there is still a hole.

If the multiplicity of the zero in the denominator was greater than the numerator, then there would not be a hole at x=3, instead there would be a vertical asymptote at x=3.