**4.8B Vectors**

AP Precalculus

# 4.8B Practice Solutions

**Instructions: Find the unit vector for the given vector.**

1) $\langle 3, 4 \rangle$

$\|v\| = \sqrt{3^2 + 4^2} = 5$

$\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

2) $\langle 10, -4 \rangle$

$\|v\| = \sqrt{10^2 + (-4)^2} = \sqrt{116} = 2\sqrt{29}$

$\left\langle \frac{10}{2\sqrt{29}}, \frac{-4}{2\sqrt{29}} \right\rangle$

$\left\langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right\rangle$

3) $\langle -5, 8 \rangle$

$\|v\| = \sqrt{(-5)^2 + (8)^2} = \sqrt{89}$

$\left\langle \frac{-5}{\sqrt{89}}, \frac{8}{\sqrt{89}} \right\rangle$

**Instructions: Find the dot product for the following vectors.**

4) $\langle 2, 4 \rangle$ and $\langle 3, 9 \rangle$

$2 \cdot 3 + 4 \cdot 9$

$6 + 36$

$42$

5) $\langle -4, 8 \rangle$ and $\langle -3, -8 \rangle$

$-4(-3) + 8(-8)$

$12 + -64$

$-52$

6) $\langle 5, 2 \rangle$ and $\langle 4, 10 \rangle$

$5(4) + 2(10)$

$20 + 20$

$40$

**Instructions: Find the angle between the two vectors.**

7) $\langle -3, -5 \rangle$ and $\langle -15, 9 \rangle$

$-3(-15) + -5(9)$

$45 + -45 = 0$

$\|a\| = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}$

$\|b\| = \sqrt{(-15)^2 + (9)^2} = \sqrt{306}$

$\|a\| \cdot \|b\| \cdot \cos\theta = 0$

$\sqrt{34} \cdot \sqrt{306} \cdot \cos\theta = 0$

$\cos\theta = 0$

$90^\circ$

8) $\langle 2, 4 \rangle$ and $\langle -1, 8 \rangle$

$2(-1) + (4)(8) = 30$

$\|a\| = \sqrt{2^2 + 4^2} = \sqrt{20}$

$\|b\| = \sqrt{(-1)^2 + 8^2} = \sqrt{65}$

$\|a\| \cdot \|b\| \cos\theta = 30$

$\sqrt{20} \cdot \sqrt{65} \cos\theta = 30$

$\sqrt{1300} \cos\theta = 30$

$\cos\theta = \frac{30}{\sqrt{1300}}$

$\theta = \cos^{-1}\left(\frac{30}{\sqrt{1300}}\right) = 41.4^\circ$

9) $\langle 5, -1 \rangle$ and $\langle 4, 6 \rangle$

$5(4) + (-1)(6) = 14$

$\|a\| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$

$\|b\| = \sqrt{4^2 + 6^2} = \sqrt{52}$

$\|a\| \|b\| \cos\theta = 14$

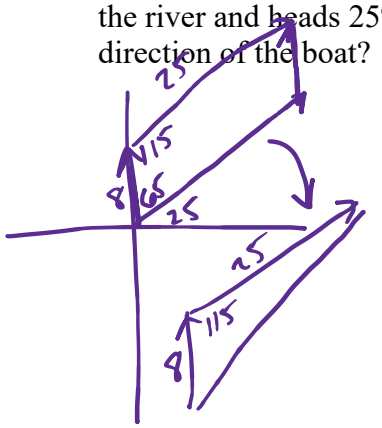
$\sqrt{26}\sqrt{52} \cos\theta = 14$

$\cos\theta = \frac{14}{\sqrt{1352}}$

$\theta = 67.6^\circ$

**Instructions: Use the Law of Sines and Cosines to solve the following.**

10) A river flows directly north with a current that is 8 mph. A ferry boat leaves the west edge of the river and heads 25° north of east at a speed of 20 mph. What is the actual speed and direction of the boat?




$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 20^2 + 8^2 - 2(20)(8)\cos 115$

$c^2 = 464 - 320\cos 115$

$c^2 = 599.24$

$c = 24.5$

$\frac{\sin\theta}{8} = \frac{\sin 115}{24.5}$

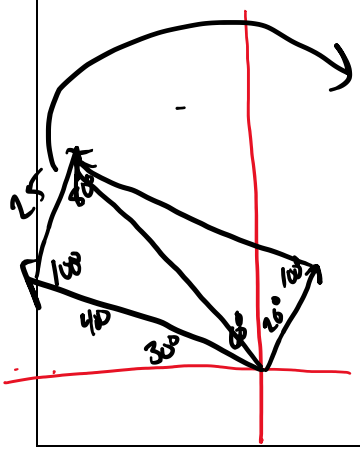
$\sin\theta = \frac{8\cdot\sin 115}{24.5}$

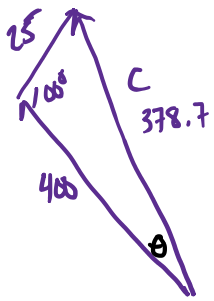
$17.2^\circ = \theta = \sin^{-1}\left(\frac{8\cdot\sin 115}{24.5}\right)$

so $25 + 17.1$

$42.2^\circ$ NORTH OF EAST

11) An airplane takes off in the direction of 30° north of west at a speed of 400 mph. The wind current is blowing at 20° east of north at a speed of 25 mph. What is the ground speed and direction of the plane?




$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 25^2 + 400^2 - 2(400)(25)\cos 100$

$c^2 = 160625 - 20000\cos 100$

$c^2 = 143378.6224$

$c = 378.7$ mph

$\frac{\sin\theta}{25} = \frac{\sin 100}{378.7}$

$\sin\theta = \frac{25\cdot\sin 100}{378.7}$

$\theta = \sin^{-1}\left(\frac{25\cdot\sin 100}{378.7}\right)$

$\theta = 3.7^\circ$

$30 + \theta = 30 + 3.7$

$= 33.7^\circ$ NORTH OF WEST

## 4.8B Vectors

**4.8B Test Prep**

12. (1.7A) Given $f(x) = x^2 + a^2$ and $g(x) = x^2 - a^2$ where $a$ is a constant integer. The function $r(x) = \frac{f(x)}{g(x)}$. What is the domain of $r(x)$ ?

(A) $(-\infty, -a) \cup (a, \infty)$

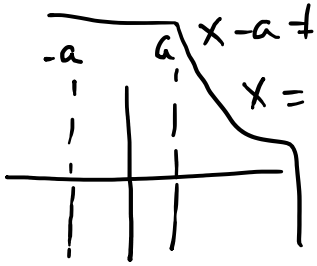
(B) $(-a, a)$

(C) $(-\infty, -a) \cup (-a, a) \cup (a, \infty)$

(D) $(-\infty, -a)$

(E) $(a, \infty)$

$\frac{x^2 + a^2}{x^2 - a^2} = \frac{x^2 + a^2}{(x-a)(x+a)}$




$x - a \neq 0$ $\quad x + a \neq 0$

$x = a$ or $-a$

13. (1.7B) The function $f$ is given by $f(x) = \frac{ax^3-2x^2+5}{2x^3-8}$ and has line $y = 3$ as a horizontal asymptote. Which of the following must be true?

(A) $f(a) = 6$

(B) $a = 6$

(C) $\lim_{x\to\infty} f(x) = a$

(D) $\lim_{x\to\infty} f(x) = 6$

(E) None of the above are true.

Divide leading coefficients if degree of numerator and denominator are the same to find horizontal asymptote.

so $\frac{a}{2} = 3$

$a = 6$

14. (1.8) The function $f$ is given by $f(x) = \frac{x^2+2x-24}{4-x}$. Which of the following describes the function $f$ ?

(A) The graph of $f$ has an $x$-intercept at $x = -6$ and a vertical asymptote of $x = 4$.

(B) The graph of $f$ has an $x$-intercept at $x = -6$ and a hole at $x = 4$.

(C) The graph of $f$ has an $x$-intercept at $x = -6$ and a vertical asymptote of $x = -4$.

(D) The graph of $f$ has an $x$-intercept at $x = -6$ and a hole at $x = -4$.

(E) The graph of $f$ has $x$-intercepts at $x = -6$ and $x = 4$.

y int: $y = \frac{0^2+2(0)-2}{4-0}$

$y = \frac{-24}{4}$

$y = -6$

$(0, -6)$

$f(x) = \frac{(x+6)(x-4)}{-(x-4)}$ hole at $x = 4$

x int: $0 = (x+6)(x-4)$

$0 = x+6$ $(-6, 0)$

$-6 = x$