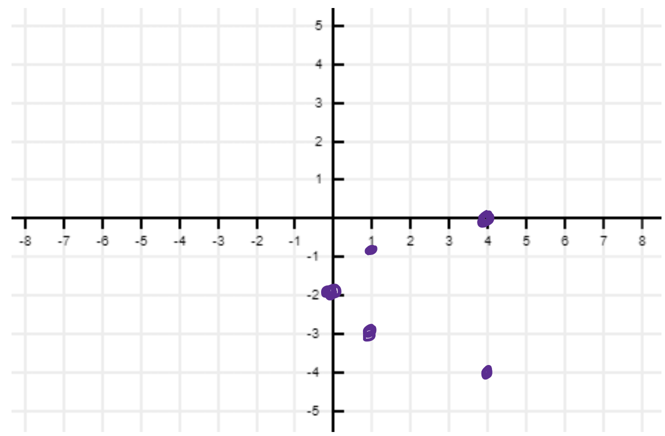
4.9 Vector-Valued Functions

4.9 Practice Solutions

Directions: For the given vector-valued functions, complete the table and sketch the graph that the endpoints make.

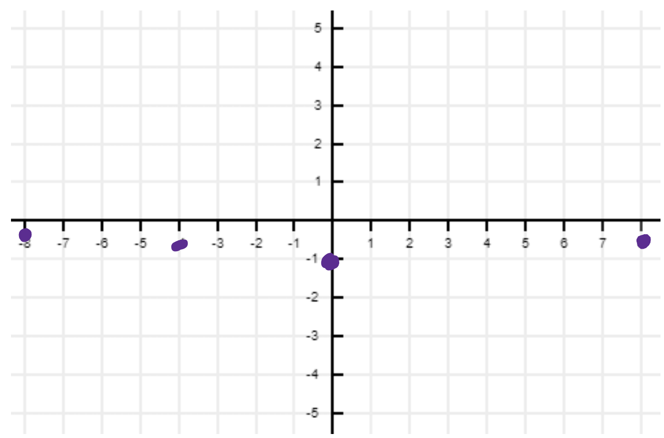
1) $f(t) = \langle t^2, t-2 \rangle$.

t	x	y
-2	4	-4
-1	1	-3
0	0	-2
1	1	-1
2	4	0



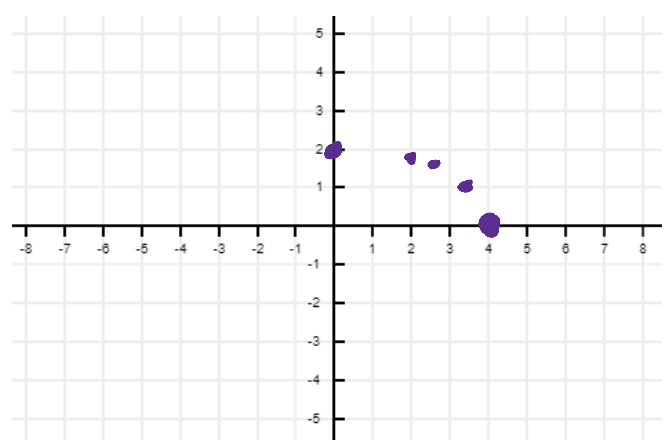
2) $f(t) = \left\langle t+4, \frac{4}{t} \right\rangle$

t	x	y
-12	-8	$-1/3$
-8	-4	$-1/2$
-4	0	-1
0	4	DNE
4	8	$1/2$



3) $f(t) = \langle 4\cos t, 2\sin t \rangle$

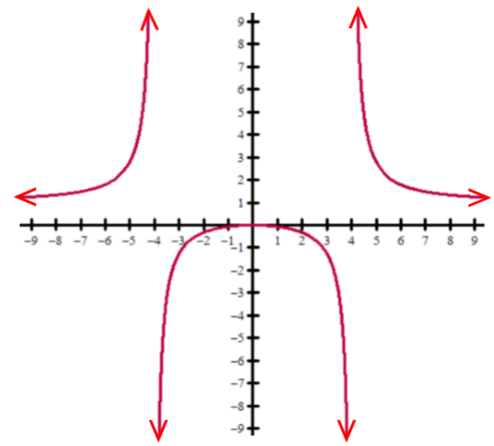
t	x	y
0	4	0
$\frac{\pi}{6}$	$2\sqrt{3} \approx 3.46$	1
$\frac{\pi}{4}$	$2\sqrt{2} \approx 2.8$	$\sqrt{2} \approx 1.4$
$\frac{\pi}{3}$	2	$\sqrt{3} \approx 1.73$
$\frac{\pi}{2}$	0	2



Directions: Find the domains of each of the components of the vector-valued function, then find the domain of the vector-valued function.	
4) $f(t) = \langle 3t-4, \frac{3}{t} \rangle$ X Comp Domain: $\mathbb{R}$ Y comp Domain: $(-\infty, 0) \cup (0, \infty)$ Function Domain: $(-\infty, 0) \cup (0, \infty)$	5) $f(t) = \langle \sqrt{x} - 3, \frac{t}{t-5} \rangle$ X Comp Domain: $(0, \infty)$ Y comp Domain: $(-\infty, 5) \cup (5, \infty)$ Function Domain: $(0, 5) \cup (5, \infty)$
6) $f(t) = \langle 2t^3, \lvert t \rvert \rangle$ X Comp Domain: $\mathbb{R}$ Y comp Domain: $\mathbb{R}$ Function Domain: $\mathbb{R}$	7) $f(t) = \langle \sqrt{t-3}, \frac{2t}{t-6} \rangle$ X Comp Domain: $(3, \infty)$ Y comp Domain: $(-\infty, 6) \cup (6, \infty)$ Function Domain: $(3, 6) \cup (6, \infty)$

Directions: Describe the motion and find the speed of a particle in motion with the following vector at the given time.	
8) $v(t) = \langle -4\cos t, 8\sin t \rangle, t = \frac{\pi}{4}$ $\langle -4\cos\frac{\pi}{4}, 8\sin\frac{\pi}{4} \rangle$ $\langle -4\left(\frac{\sqrt{2}}{2}\right), 8\left(\frac{\sqrt{2}}{2}\right) \rangle$ $\langle -2\sqrt{2}, 4\sqrt{2} \rangle$ @ $t = \frac{\pi}{4}$ it is moving left and up. $\lVert v \rVert = \sqrt{(-2\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{8+32} = \sqrt{40}$	9) $v(t) = \langle 2(t-3)^2, \sqrt{2t-3} \rangle, t = 6$ $\langle 2(6-3)^2, \sqrt{2(6)-3} \rangle$ $\langle 18, 3 \rangle$ $\lVert v \rVert = \sqrt{18^2 + 3^2}$ $= \sqrt{333} \approx 18.25$ ITS moving right and up at this speed
10) $v(t) = \langle \frac{5t-4}{t}, 2t^3 \rangle, t = -2$ $\langle \frac{5(-2)-4}{(-2)}, 2(-2)^3 \rangle = \langle 7, -16 \rangle$ $\lVert v \rVert = \sqrt{7^2 + (-16)^2} = \sqrt{305} \approx 17.46$ ITS MOVING RIGHT AND DOWN AT THIS SPEED	11) $v(t) = \langle \frac{-t^2-5}{t}, -\sqrt{t-4} \rangle, t = 5$ $\langle \frac{-5^2-5}{5}, -\sqrt{5-4} \rangle$ $\langle -6, -3 \rangle$ $\lVert v \rVert = \sqrt{(-6)^2 + (-3)^2}$ $= \sqrt{36+9} = \sqrt{45} \approx 6.7$ IT IS MOVING LEFT AND DOWN AT THIS SPEED

12. (1.9) Given the graph of f. Which of the following describes the function



(A) $\lim_{x\to -4^-} f(x) = -\infty$ and $\lim_{x\to -4^+} f(x) = -\infty$

(B) $\lim_{x\to -4^-} f(x) = \infty$ and $\lim_{x\to -4^+} f(x) = -\infty$

(C) $\lim_{x\to -4^-} f(x) = -\infty$ and $\lim_{x\to -4^+} f(x) = \infty$

(D) $\lim_{x\to -4^-} f(x) = \infty$ and $\lim_{x\to -4^+} f(x) = \infty$

(E) $\lim_{x\to -4} f(x) = f(0)$

12. (1.10) The figure shows the graph of a function f. Which of the following could be an expression for the $f(x)$?

hole @ $x=4$

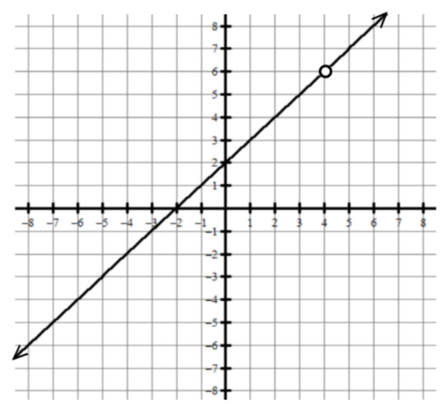
so $\frac{\cancel{(x-4)}}{\cancel{(x-4)}}$

(A) $\frac{(x+2)(x-4)}{(x+2)}$

(B) $\frac{(x-2)(x+4)}{(x-2)}$

(C) $\frac{(x+2)(x-4)}{(x-4)}$

(D) $\frac{(x-2)(x+4)}{(x+4)}$



graph of *f*