1.11A Equivalent Representations and Binomial Theorem

AP Precalculus

1.11A Practice

The following are in factored form. Convert to standard form and answer the questions.

1. $f(x) = 2(x-4)^2$ $2(x-4)(x-4)$ $2[x^2-4x-4x+16]$

   a. Standard Form: $2x^2-16x+32$

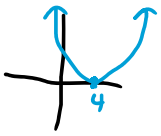
   b. Degree: 2 ↑↑

   c. End Behavior: $\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = \infty$

   d. Zero(s): $x = 4$

   e. Where is $f(x) \geq 0$?

   $(-\infty, \infty)$

   f. y-intercept: 32

2. $h(x) = \frac{2(x+3)(x-3)}{(x+2)(x+3)}$ $\frac{2[x^2+3x-3x-9]}{x^2+3x+2x+6}$

   a. General Form: $\frac{2x^2-18}{x^2+5x+6}$

   b. Domain: $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

   c. Zero(s): $x = 3$

   d. Hole(s): $x = -3$

   e. Vertical Asymptote(s): $x = -2$

   f. Horizontal Asymptote: $y = 2$

   g. y-intercept: $\frac{-18}{6} = -3$

The following are in standard form. Convert to factored form and answer the questions.

3. $g(x) = -3x^3 + 27x$

   $-3x(x^2-9)$

   a. Factored Form: $-3x(x+3)(x-3)$

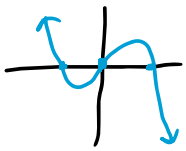
   b. Degree: 3 ↑↓

   c. End Behavior: $\lim_{x\to\infty} f(x) = -\infty$ $\lim_{x\to-\infty} f(x) = \infty$

   d. Zero(s): $x = 0, -3, 3$

   e. Where is $f(x) \geq 0$?

   $(-\infty,-3)\cup(0,3)$

   f. y-intercept: 0

4. $h(t) = \frac{t^3-4t^2}{t^2+2t-15}$ $\frac{t^2(t-4)}{(t+5)(t-3)}$

   a. Factored Form: ↗

   b. Domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$

   c. Zero(s): $x = 0$ and 4

   d. Hole(s): none

   e. Vertical Asymptote(s): $x = -5$ and 3

   f. Horizontal Asymptote: none

   g. y-intercept: $\frac{0}{-15} = 0$

Use the binomial theorem to expand the following.

5. $(x+2)^4$

$1(x)^4 + 4(x)^3(2)^1 + 6(x)^2(2)^2 + 4(x)^1(2)^3 + 1(2)^4$

$x^4 + 8x^3 + 24x^2 + 32x + 16$

1
1 1
1 2 1
1 3 3 1
→ 1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

6. $(x-3)^5$

$1(x)^5 + 5(x)^4(-3)^1 + 10(x)^3(-3)^2 + 10(x)^2(-3)^3 + 5(x)^1(-3)^4 + 1(-3)^5$

$x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
→ 1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

7. $(2x+1)^3$

$1(2x)^3 + 3(2x)^2(1)^1 + 3(2x)^1(1)^2 + 1(1)^3$

$8x^3 + 12x^2 + 6x + 1$

1
1 1
1 2 1
→ 1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

8. $(3x-2)^4$

$1(3x)^4 + 4(3x)^3(-2)^1 + 6(3x)^2(-2)^2 + 4(3x)^1(-2)^3 + 1(-2)^4$

$81x^4 - 216x^3 + 216x^2 - 96x + 16$

1
1 1
1 2 1
1 3 3 1
→ 1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Find the given term in the binomial expansion.

9. $(x+5)^5$

Find the 3rd term.

$10(x)^3(5)^2$

$250x^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
→ 1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

10. $(x-2)^7$

Find the 4th term.

$35(x)^4(-2)^3$

$-280x^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
→ 1 7 21 35 35 21 7 1

11. $(3x-1)^8$

Find the 5th term.

$70(3x)^4(-1)^4$

$5670x^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
→ 1 8 28 56 70 56 28 8 1

12. $(2x+3)^4$

Find the last term.

$1(2x)^0(3)^4$

81

1
1 1
1 2 1
1 3 3 1
→ 1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Multiple Choice

13. What is the leading coefficient of the fourth term when $(x-2)^6$ is expanded?

(A) -160
(B) -120
(C) 120
(D) 160

$20(x)^3(-2)^3$

$20(x^3)(-8)$

$-160x^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
→ 1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

14. What is the second term when $(2a-b)^4$ is expanded?

(A) $-32a^3b$
(B) $-4a^2b^2$
(C) $-16ab^3$
(D) $-ab^4$

$4(2a)^3(-b)^1$

$4(8a^3)(-b)$

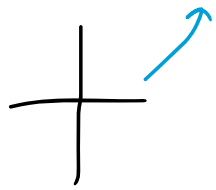
$-32a^3b$

1
1 1
1 2 1
1 3 3 1
→ 1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

15. In the xy-plane, the graph of a function f has $\lim_{x\to\infty} f(x) = \infty$. Which of the following could be an expression $f(x)$?

(A) $\frac{(x+3)^3}{x(x^3+1)}$ nope horizontal asymptote $y=0$
(B) $-2(x-1)^3(x+5)$ nope 4th degree, negative
(C) $\frac{x(x+3)^3}{x(x^3+1)}$ nope horizontal asymptote $y=1$
(D) $x(x+2)^3(x-1)$ YES! 5th degree, positive

$\lim_{x\to\infty} f(x) = \infty$



As x approaches infinity the f(x) approaches infinity

So, the graph of f must go up as x goes right!