1.14 Function Model Construction

AP Precalculus

1.14 Practice

CALCULATOR ACTIVE Perform the indicated regression and answer the questions.

1. Use Quadratic Regression.

x	1	2	3	4	5	6
$f(x)$	10	6	5	5.5	6	11

a. Write the equation of the regression curve.

$f(x) = 0.911x^2 - 6.218x + 15.2$

b. Use your equation to predict $f(2.5)$.

$f(2.5) = 0.911(2.5)^2 - 6.218(2.5) + 15.2$

$f(2.5) = 5.349$

2. Use Quartic Regression.

x	-6	-4	-2	2	4	6
$g(x)$	2700	928	80	66	500	2450

a. Write the equation of the regression curve.

$g(x) = 1.239x^4 + 0.667x^3 + 28.646x^2 - 47.595x - 61.4$

b. Use your equation find average rate of change from $x = 3$ to $x = 6$.

$g(3) = 171.997$

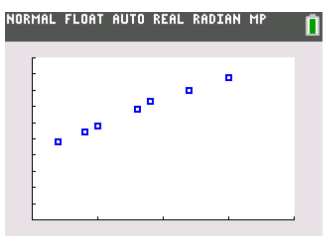
$g(6) = 2434.1$

$\dfrac{2434.1 - 171.997}{6 - 3} = \dfrac{2262.103}{3} = 754.034$

CALCULATOR ACTIVE Graph the data and choose the regression that best fits the data.

3. The data shows the salary in thousands of dollars for employees given their years experience.

Experience (years)	Salary (thousands)
2	48
4	54
5	58
8	68
9	73
12	80
15	88



a. Is the data Linear, Quadratic, or Cubic ?

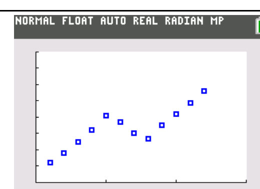
b. Write the equation of the regression curve.

$f(x) = 3.137x + 42.349$

c. Use your equation predict the salary of an employee with 11 years experience.

$f(11) = 76.856$ thousand dollar salary

4. The data shows poll ratings of a politician over time.



month	1	2	3	4	5	6	7	8	9	10	11	12
rating	12	18	25	32	41	37	30	27	35	42	49	56

a. Is the data Linear, Quadratic, or Cubic?

b. Write the equation of the regression curve.

$f(x) = 0.161x^3 - 3.130x^2 + 19.849x - 7.364$

c. Use your equation to find the average rate of change from May to October.

$g(5) = 33.756$

$g(10) = 39.126$

$\frac{33.756 - 39.126}{5-10} = \frac{-5.37}{-5} = 1.074$

increase in rating every month

CALCULATOR ACTIVE Construct an inversely proportional model to answer the following.

5. The time taken for a water heater to boil water is inversely proportional to the power used in watts. It takes 2000 watts to boil water in 240 seconds. Find the time it takes when using 900 watts.

$t = \frac{k}{w}$

$2000 \cdot 240 = \frac{k}{2000} \cdot 2000$

$k = 480000$

$t = \frac{480000}{900}$

$\boxed{t = 533.\overline{3} \text{ seconds}}$

6. The data is inversely proportional. Find the value of *a*.

$\boldsymbol{x}$	3	10	12
$\boldsymbol{f(x)}$	16	4.8	a

$y = \frac{k}{x}$

$3 \cdot 16 = \frac{k}{3} \cdot 3$

$k = 48$

$y = \frac{48}{12}$

$y = 4$

$\boxed{a = 4}$

7. Price is inversely proportional to time squared. If on day 2 the price is \$4, what is the price on day 4?

$p = \frac{k}{t^2}$

$4 \cdot 4 = \frac{K}{2^2} \cdot 4$

$16 = K$

$p = \frac{16}{4^2}$

$\boxed{p = 1 \text{ dollar}}$

8. The cost per cookie for making cookies is inversely proportional to the square root of the number of cookies made. If it costs \$2 each to make 9 cookies, how much would it cost for each cookie to make 25 cookies?

$c = \frac{k}{\sqrt{n}}$

$3 \cdot 2 = \frac{K}{\sqrt{9}} \cdot 3$

$6 = k$

$c = \frac{6}{\sqrt{25}} = \frac{6}{5}$

$\boxed{c = 1.20 \text{ dollars}}$

CALCULATOR ACTIVE Use the piecewise function to answer the following.

9.

$$f(x) = \begin{cases} 9x - x^2, & -2 \le x \le 3 \\ 2x - 17, & 3 < x < 10 \\ 5, & 10 \le x \le 12 \end{cases}$$

a. Find $f(1)$.

$f(1) = 9(1) - (1)^2 = 8$

b. Find $f(10)$.

$f(10) = 5$

10. A rental car costs \$20 to rent plus 70 cents every mile driven for the first 100 miles. Miles driven over 100 only cost 40 cents.

$$C(m) = \begin{cases} 20 + 0.7m, & 0 \le m \le 100 \\ 0.4(m - 100) + 90, & m > 100 \end{cases}$$

a. Find $C(128)$.

$f(128) = 0.4(128 - 100) + 90 = 101.2$

b. How many miles did a customer drive that spent 24 dollars on their rental car?

$24 = 20 + 0.7m$

$\boxed{m = 5.714 \text{ miles}}$

OR $24 = 0.4(m - 100) + 90$

$\cancel{m = -165 \text{ miles}}$

can't be negative

Multiple Choice – Calculator Active

11. The table below shows the average price of a movie ticket during certain years.

t	1	4	7	8	9	11
year	2012	2015	2018	2019	2020	2022
price	7.96	8.17	8.97	9.11	9.16	10.12

A linear regression is used to construct a function model P that models the price over the given years. If $t = 1$ corresponds to 2012, $t = 4$ corresponds to 2015, and this pattern continues, which of the following defines function P?

(A) $P(t) = 0.206x + 7.541$

(B) $P(t) = 0.397x + 7.524$

(C) $P(t) = 0.206x + 7.747$

(D) $P(t) = 0.206x - 407.07$

---

12. The weight of an object is inversely proportional to the square of the distance between an object and the center of the earth. This relationship is modeled by the function W, where $W(d) = \frac{2.944\times10^9}{d^2}$ for distance, d, measured in feet, and weight where $W(d)$ measured in pounds. What is the average rate of change, in pounds per foot, if the distance between an object and the center of the earth is increased from 8500 feet to 9500 feet?

(A) 2944

(B) 8.127

(C) -123.047

(D) -0.00817

$W(8500) = 40.747$

$W(9500) = 32.62$

$$\frac{32.62 - 40.747}{9500 - 8500}$$

---

13. A membership to World Fitness costs \$75 per month and includes 10 free fitness classes. Any fitness classes attended after the first 10 free fitness classes cost \$5 each. Function C is used to model the cost of a monthly membership to World Fitness where n is the number of fitness classes taken and $C(n)$ is the cost in dollars. Which of the following defines C ?

(A) $C(n) = \begin{cases} 75, & 0 \le n \le 10 \\ 5n + 75, & n > 10 \end{cases}$

(B) $C(n) = \begin{cases} 75, & 0 \le n \le 10 \\ 5(n-10) + 75, & n > 10 \end{cases}$

(C) $C(n) = \begin{cases} 75, & 0 \le n \le 10 \\ 10(n-5) + 75, & n > 10 \end{cases}$

(D) $C(n) = 5n + 75$

FREE RESPONSE

Police use a formula to estimate the speed a car was traveling before an accident by measuring its skid marks. Function S is used to model the speed the car was traveling in mph where d is the distance the car skidded in feet and f is the coefficient of friction which depends on the road surface and road conditions.

$$S(d) = \sqrt{30df}$$

a. A country road has a coefficient of friction of 0.9 when it is dry and 0.4 when it is wet. What values of distance skidded would you expect in both conditions for a car that was travelling 110 mph when the brakes were applied?

$$110 = \sqrt{30d(0.9)} \qquad 110 = \sqrt{30d(0.4)}$$
$$d = 448.148\ feet \qquad d = 1008.333\ feet$$

b. Analyze the rate of change from a measured 70-foot skid mark to a 160-foot skid mark on the dry country road.

$$S(70) = \sqrt{30(70)(0.9)}$$
$$S(160) = \sqrt{30(160)(0.9)}$$

$\frac{65.727-43.474}{160-70} = \frac{22.253}{90} = 0.247$ mph per foot of skid mark

c. The distance of a skid mark is inversely proportional to the product of 30 and the coefficient of friction. Using the 60-foot skid mark on a dry country road, find the constant of proportionality. Explain what it means in this context.

$$d = \frac{k}{30f}$$
$$60 = \frac{k}{30(0.9)}$$
$$k = 1620$$

$$S = \sqrt{30d(0.9)}$$
$$S^2 = 30d(0.9)$$
$$S^2 = 30d(0.9)$$
$$\frac{S^2}{30(0.9)} = d$$

$$\frac{k}{30(0.9)} = \frac{S^2}{30(0.9)}$$

$k\ is\ speed^2$ so the speed is $\sqrt{k}$