

1.3 Rates of Change in Linear and Quadratic Functions

AP Precalculus

1.3 Practice

What is the average rate of change for each function on the given intervals?

1. $y = 7x + 10$ on $-6 \leq x \leq -4$

The average rate of change of a linear function is constant. In this case, it is 7.

2. $y = 5x - 3$ on $1 \leq x \leq 7$

The average rate of change of a linear function is constant. In this case, it is 5.

3. $y = x^2 + 4x - 2$ on $-3 \leq x \leq 2$

$$y(2) = (2)^2 + 4(2) - 2 = 4 + 8 - 2 = 10$$

$$y(-3) = (-3)^2 + 4(-3) - 2 = 9 - 12 - 2 = -5$$

$$\frac{10 - (-5)}{2 - (-3)} = \frac{15}{5} = \boxed{3}$$

4. $y = 2x^2 + 2x + 1$ on $2 \leq x \leq 5$

$$y(5) = 2(5)^2 + 2(5) + 1 = 50 + 10 + 1 = 61$$

$$y(2) = 2(2)^2 + 2(2) + 1 = 8 + 4 + 1 = 13$$

$$\frac{61 - 13}{5 - 2} = \frac{48}{3} = \boxed{16}$$

For each problem, a quadratic function g has the given average rates of change on an interval.

5. Avg rate of change is 1 on the interval $-3 \leq x \leq 0$.
Avg rate of change is 5 on the interval $0 \leq x \leq 3$.
What is the average rate of change of g on the interval $3 \leq x \leq 6$?

$\boxed{9}$

The rate of change of the average rate of change of a quadratic function is constant for equal length intervals. For this problem, the avg rate of change is increasing by 4 for each interval of 3.

6. Avg rate of change is -1 on the interval $1 \leq x \leq 5$.
Avg rate of change is -7 on the interval $5 \leq x \leq 9$.
What is the average rate of change of g on the interval $17 \leq x \leq 21$?

$\boxed{-25}$

The avg rate of change is decreasing by 6 for each interval of 4. For $9 \leq x \leq 13$, the avg rate of change is -13 . For $13 \leq x \leq 17$, the avg rate of change is -19 . For $17 \leq x \leq 21$, the avg rate of change is -25 .

7. Avg rate of change is 2 on the interval $-1 \leq x \leq 1$.
 Avg rate of change is 7 on the interval $1 \leq x \leq 3$.
 What is the average rate of change of g on the interval $-3 \leq x \leq -1$?

-3

The avg rate of change is increasing by 5 for each interval of 2, but we must go "back" to the interval $-3 \leq x \leq -1$ and decrease by 5.

8. Avg rate of change is -6 on the interval $0 \leq x \leq 5$.
 Avg rate of change is -2 on the interval $5 \leq x \leq 10$.
 What is the average rate of change of g on the interval $20 \leq x \leq 25$?

10

The avg rate of change is increasing by 4 for each interval of 5. For $10 \leq x \leq 15$, the avg rate of change is 2. For $15 \leq x \leq 20$, the avg rate of change is 6. For $20 \leq x \leq 25$, the avg rate of change is 10.

What is the rate of change of the average rates of change for each function over consecutive equal-length intervals?

9. $y = 13x - 10$

The average rate of change of the rates of change of a linear function is always **ZERO**.

10. $f(x) = x^2 + 3x + 8$

$f(-1) = 1 - 3 + 8 = 6$
 $f(0) = 8$
 $f(1) = 1 + 3 + 8 = 12$
 $f(2) = 4 + 6 + 8 = 18$

$> +2$
 $> +4$
 $> +6$

Rate of change is changing by 2.

2

11. $f(x) = 6x - 2x^2 + 1$

$f(-1) = -6 - 2 + 1 = -7$
 $f(0) = 1$
 $f(1) = 6 - 2 + 1 = 5$
 $f(2) = 12 - 8 + 1 = 5$

$> +8$
 $> +4$
 $> +0$

Rate of change is changing by -4 .

-4

The values of a function are given at selected x -values in the table below. The function's concavity does not change. Determine if the function is concave up or concave down. Justify your answer.

12.

x	-8	-6	-4	-2	0
$g(x)$	10	11	14	20	29

\downarrow \downarrow \downarrow \downarrow
 $+1$ $+3$ $+6$ $+9$

Concave up because the rate of change is increasing over equal-length input-value intervals.

13.

x	5	10	15	20	25
$h(x)$	100	75	40	0	-50

\downarrow \downarrow \downarrow \downarrow
 -25 -35 -40 -50

Concave down because the rate of change is decreasing over equal-length input-value intervals.

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1.3 Test Prep

14. The table gives values of the function h for selected values of x .

x	-8	-4	4	
$h(x)$	23	17	5	

11 more changes

$$5 + (11)\left(-\frac{3}{2}\right)$$

$$5 - \frac{33}{2}$$

$$\frac{10}{2} - \frac{33}{2}$$

$$\left(-\frac{23}{2}\right)$$

If the function h is linear, what is the value of $h(15)$?

(A) $-\frac{21}{2}$

(B) $-\frac{23}{2}$

(C) -5

(D) -11

Slope = $\frac{17 - 23}{-4 - (-8)} = \frac{-6}{4} = -\frac{3}{2}$

or $\frac{5 - 17}{4 - (-4)} = \frac{-12}{8} = -\frac{3}{2}$

B

15. The table gives values of the function g for selected values of x .

x	0	2	4	6	8
$g(x)$	-5	-1	0	-2	-7

If the function g is quadratic, what is the value of $g(8)$?

(A) -5

(B) 5

(C) -7

(D) 7

+4 +1 -2 -5
 -3 rate of change is decreasing by 3 for every 2 x -values.

C

16. The function p is given by $p(x) = g(x + 1) - g(x)$. If $p(x) = 2$, which of the following statements must be true?

D

- I. Because p is positive and constant, the graph of g always has positive slope. ✓
- II. Because p is positive and constant, the graph g is concave up. ✗
- III. Because p is positive and constant, g is increasing. ✓

(A) I only

(B) II only

(C) III only

(D) I and III only

The rate of change of $g(x)$ is $\frac{g(x+1)-g(x)}{x+1-x} = g(x + 1) - g(x) = p(x)$.

Therefore, $p(x)$ is the rate of change of $g(x)$, which equals 2. A constant rate of change would make $g(x)$ a line.