AP Precalc

Write your questions and thoughts here!

# 1.7A Rational Functions & End Behavior

**1.7A Notes**

## Rational Function

The ratio of two polynomials where the polynomial in the denominator cannot equal 0.

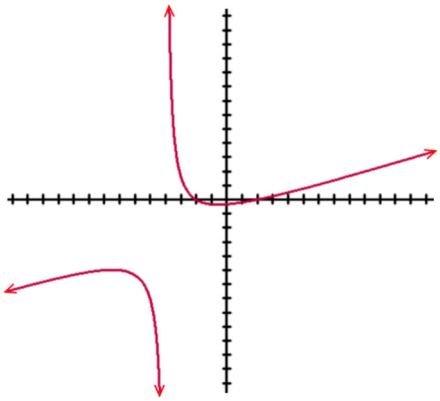
$$r(x) = \frac{f(x)}{g(x)} \text{ where } f \text{ and } g \text{ are polynomials and } g(x) \neq 0$$

## Domain of Rational Functions

**Example 1:**

1. $f(x) = \frac{x^2-4}{3x+12}$

Domain:



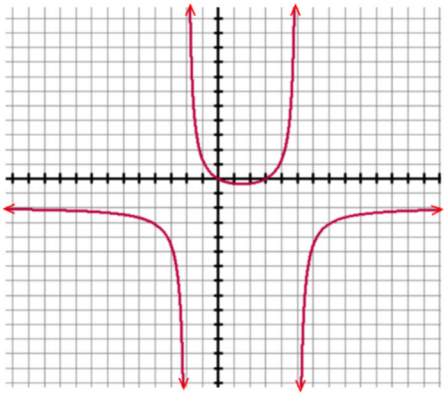
**Find the domain of the rational functions.**

2. $g(x) = \frac{4x-1}{x^2-2x-24}$

Domain:

3.

Domain:

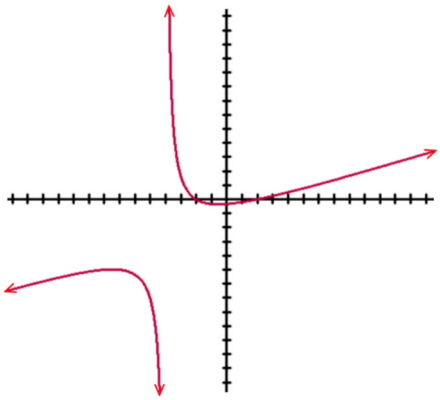


## End Behavior of Rational Functions

**Example 4:**

4. $f(x) = \frac{x^2-4}{3x+12}$

End Behavior:



Write your questions and thoughts here!

**Determine the end behavior of the following rational functions.**

5. $g(x) = \frac{4x-1}{x^2-2x-24}$

As $x$ increases without bound the $g(x)$…

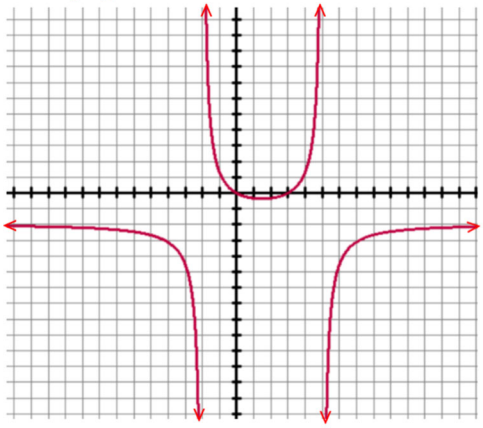
| $x$ | $g(x)$ |
|---|---|
| 10 | |
| 100 | |
| 1,000 | |
| 10,000 | |

As $x$ decreases without bound the $g(x)$…

| $x$ | $g(x)$ |
|---|---|
| $-10$ | |
| $-100$ | |
| $-1{,}000$ | |
| $-10{,}000$ | |

End Behavior:

6. The graph of $f$



End Behavior:

## Horizontal Asymptote

The line $y = b$ is a horizontal asymptote of the graph of $f$ when…

$$\lim_{x\to-\infty} f(x) = b \qquad \text{OR} \qquad \lim_{x\to\infty} f(x) = b$$

## 1.7A Rational Functions and End Behavior

AP Precalculus

**1.7A Practice**

**State the domain of the following rational functions. Use interval notation.**

1. $f(x) = \frac{x+1}{2x+5}$

Domain:

2. $g(n) = \frac{n^2-4}{(n+3)(n-7)}$

Domain:

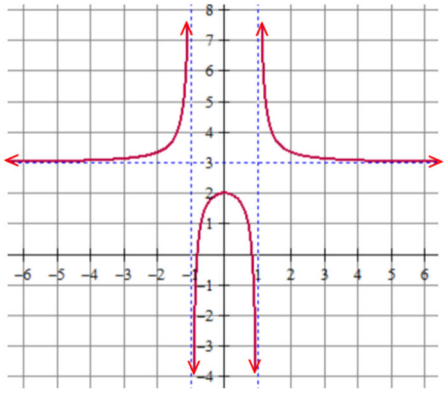
3. $h(x) = \frac{x^3+7x^2+12x}{x^2+2x-15}$

Domain:

**State the domain of the following rational functions. Use interval notation.**

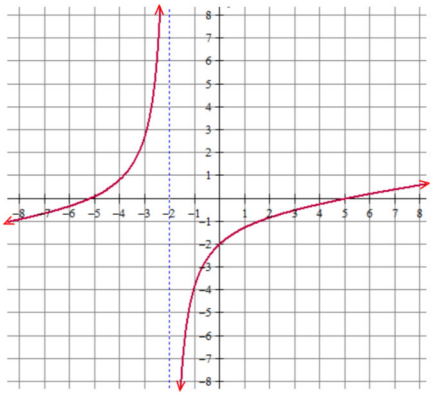
4.

Domain:



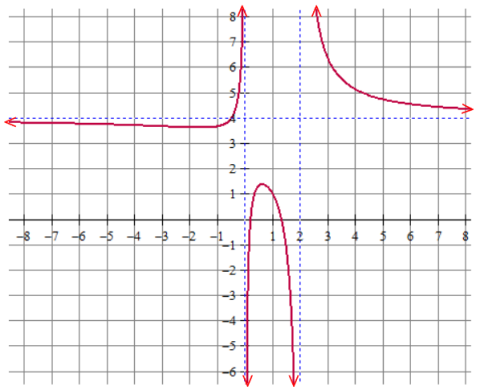
5.

Domain:



**Use the graph of the rational function $f$ to find the following.**

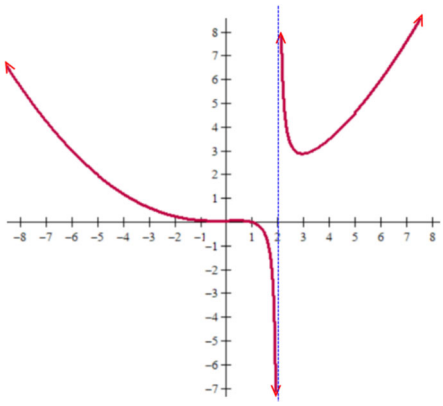
6.



End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

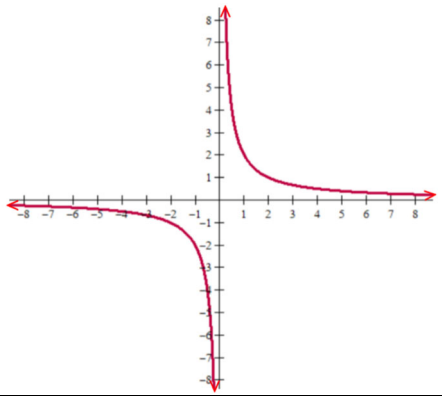
7.



End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

8.



End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

**CALCULATOR ACTIVE Complete the table to answer the following.**

9. $f(x) = \frac{x^2}{x^2-4x}$

| $x$ | $-10{,}000$ | $-1{,}000$ | $-100$ | $100$ | $1{,}000$ | $10{,}000$ |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

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10. $d(t) = \frac{t^3-3t}{2t+1}$

| $t$ | $-5{,}000$ | $-500$ | $-50$ | $50$ | $500$ | $5{,}000$ |
|---|---|---|---|---|---|---|
| $d(t)$ | | | | | | |

End Behavior:

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

**Use the table of the rational function *h* to find the following.**

11.

| $t$ | $h(t)$ |
|---|---|
| $-999{,}999$ | $4.2 \times 10^{32}$ |
| $-99{,}999$ | $7.3 \times 10^{12}$ |
| $-9{,}999$ | $6{,}524{,}378$ |
| $-999$ | $561{,}391$ |
| $0$ | $32$ |
| $999$ | $-478$ |
| $9{,}999$ | $-289{,}437$ |
| $99{,}999$ | $-2{,}198{,}235$ |
| $999{,}999$ | $-389{,}031{,}455$ |

a. Find $h(999) =$

b. Find $\lim_{t\to\infty} h(t) =$

c. As *t* increases without bound the $h(t)$…

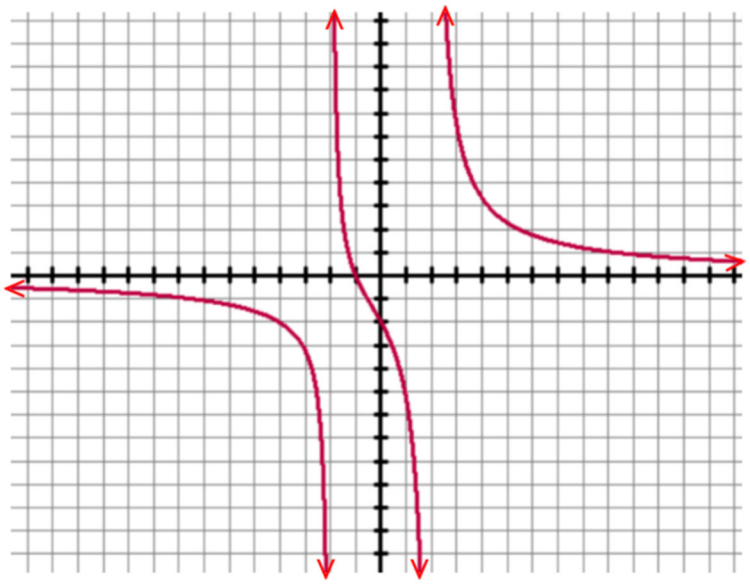
d. Find the *y*-intercept.

e. Find $\lim_{t\to-\infty} h(t) =$

f. As *t* decreases without bound the $h(t)$…

**Use the graph of the rational function $f$ to answer the following if they exist!**

12.



a. State the domain of $f(x)$.
Use interval notation.

b. Find $f(9) =$

c. Find the $y$-intercept.

d. Find the $x$-intercept(s).

e. State the interval(s) where $f(x)$ is increasing.

f. State the interval(s) where $f(x)$ is decreasing.

g. Find $\lim_{x\to\infty} f(x) =$

h. As $x$ increases without bound the $f(x)$…

i. Estimate the point of inflection.

j. Find the average rate of change over the interval $-1 \le x \le 0$.

k. Find $\lim_{x\to-\infty} f(x) =$

l. As $x$ decreases without bound the $f(x)$…

m. Find the horizontal asymptote.

n. State the interval(s) where $f(x) < 0$.

13. Which one of the following polynomials could be the denominator for the graph of the rational function shown above in question #12 ?

(A) $x^2 + 4x + 4$

(B) $x^2 - 4x + 4$

(C) $x^2 - 4$

(D) $(x - 4)^2$

(E) $(x + 4)^2$

**Multiple Choice**

14. Given $f(x) = x^2 + a^2$ and $g(x) = x^2 - a^2$ where $a$ is a constant integer. The function $r(x) = \frac{f(x)}{g(x)}$. What is the domain of $r(x)$ ?

(A) $(-\infty, -a) \cup (a, \infty)$

(B) $(-a, a)$

(C) $(-\infty, -a) \cup (-a, a) \cup (a, \infty)$

(D) $(-\infty, -a)$

(E) $(a, \infty)$

15. The function $f(x)$ is defined such that for all $x > 0$, the line $y = 4$ is a horizontal asymptote. Which of the following must be true?

(A) $f(4)$ is undefined

(B) $f(x) \neq 4$ for all $x > 0$

(C) $\lim_{x \to \infty} f(x) = 0$

(D) $\lim_{x \to \infty} f(x) = 4$

(E) $f(4) = 0$

16. Given the line $y = a$ is a horizontal asymptote of $g(x)$ for all $x < 0$, which of the following must be true?

(A) $g(a)$ is undefined

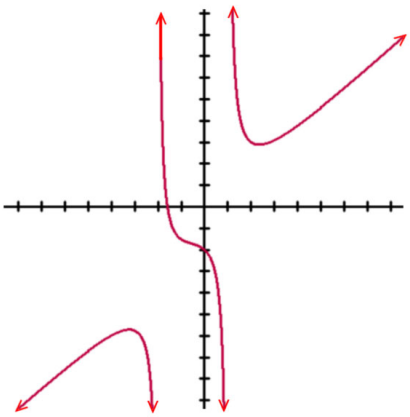
(B) $g(x) \neq a$

(C) $\lim_{x \to -\infty} g(x) = a$

(D) $\lim_{x \to -\infty} g(x) = -a$

(E) None of the above

17. Given the graph of $f$. Which describes the end behavior of $f$?



(A) $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$

(B) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

(C) $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

(D) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$

(E) $\lim_{x \to \infty} f(x) = f(0)$