1.7A Rational Functions and End Behavior

AP Precalculus

1.7A Practice

State the domain of the following rational functions. Use interval notation.

1. $f(x) = \frac{x+1}{2x+5}$

$2x+5 \neq 0$

$x \neq -\frac{5}{2}$

Domain: $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

2. $g(n) = \frac{n^2-4}{(n+3)(n-7)}$

$(n+3)(n-7) \neq 0$

$n \neq -3 \quad n \neq 7$

Domain: $(-\infty, -3) \cup (-3, 7) \cup (7, \infty)$

3. $h(x) = \frac{x^3+7x^2+12x}{x^2+2x-15}$

$(x+5)(x-3) \neq 0$

$x \neq -5 \quad x \neq 3$

Domain: $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$

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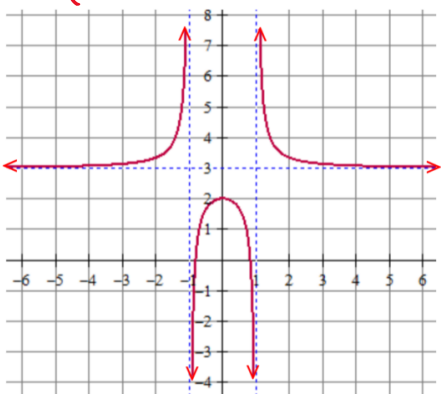
State the domain of the following rational functions. Use interval notation.

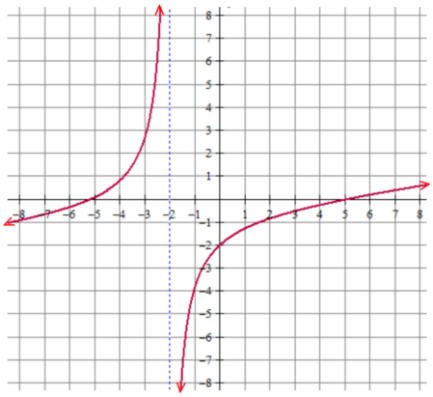
4.

Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$



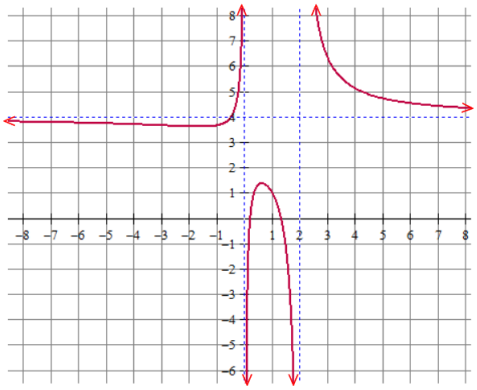
5.

Domain: $(-\infty,-2)\cup(-2,\infty)$



Use the graph of the rational function *f* to find the following.

6.



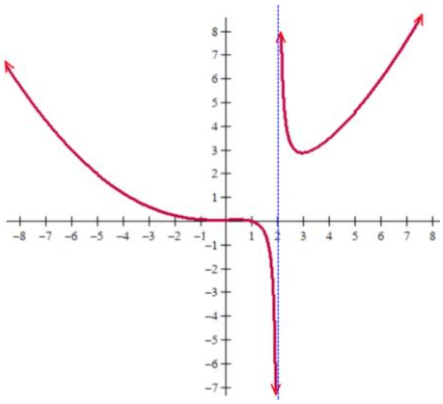
End Behavior:

$\lim_{x\to-\infty} f(x)=4$ $\lim_{x\to\infty} f(x)=4$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y=4$

7.



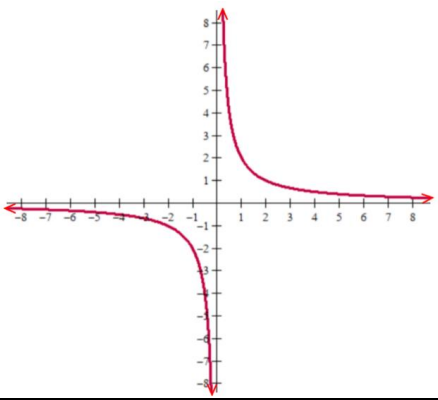
End Behavior:

$\lim_{x\to-\infty} f(x)=\infty$ $\lim_{x\to\infty} f(x)=\infty$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

no horizontal asymptote

8.



End Behavior:

$\lim_{x\to-\infty} f(x)=0$ $\lim_{x\to\infty} f(x)=0$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y=0$

CALCULATOR ACTIVE Complete the table to answer the following.

9. $f(x) = \frac{x^2}{x^2-4x}$

x	$-10{,}000$	$-1{,}000$	-100	100	$1{,}000$	$10{,}000$
$f(x)$	0.9996	0.99602	0.96154	1.0417	1.004	1.0004

End Behavior:

$\lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to \infty} f(x) = 1$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

$y = 1$

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10. $d(t) = \frac{t^3-3t}{2t+1}$

t	$-5{,}000$	-500	-50	50	500	$5{,}000$
$d(t)$	1.25×10^7	125,123	1261.1	1236.1	124,874	1.24×10^7

End Behavior:

$\lim_{t \to -\infty} d(t) = \infty$ $\lim_{t \to \infty} d(t) = \infty$

Is there a horizontal asymptote?
If so, write the equation of the horizontal asymptote.

No horizontal asymptote

Use the table of the rational function h to find the following.

11.

t	$h(t)$
$-999{,}999$	4.2×10^{32}
$-99{,}999$	7.3×10^{12}
$-9{,}999$	6,524,378
-999	561,391
0	32
999	-478
9,999	$-289{,}437$
99,999	$-2{,}198{,}235$
999,999	$-389{,}031{,}455$

a. Find $h(999) =$ -478

b. Find $\lim_{t \to \infty} h(t) =$ $-\infty$

c. As t increases without bound the $h(t)$…

approaches negative infinity

d. Find the y-intercept. $(0, 32)$

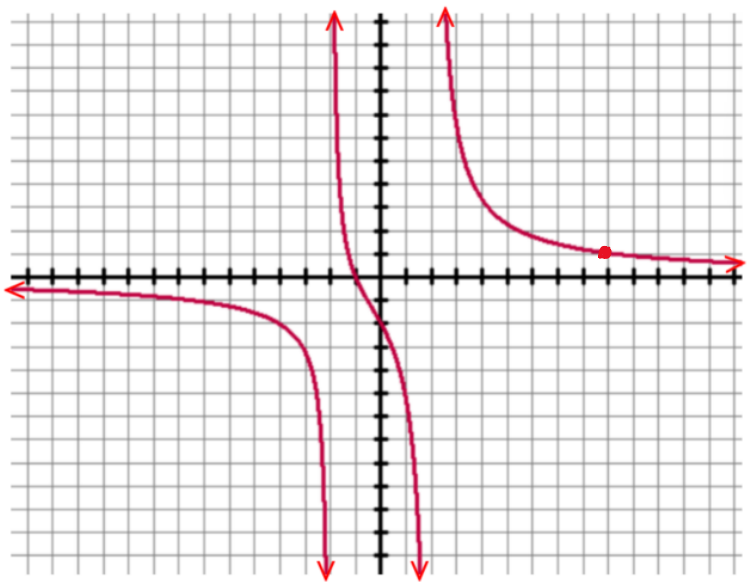
e. Find $\lim_{t \to -\infty} h(t) =$ ∞

f. As t decreases without bound the $h(t)$…

approaches infinity

Use the graph of the rational function f to answer the following if they exist!

12.



a. State the domain of $f(x)$.
Use interval notation. $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

b. Find $f(9) = 1$

c. Find the y-intercept. $(0,-2)$

d. Find the x-intercept(s). $(-1,0)$

e. State the interval(s) where $f(x)$ is increasing.

Never

f. State the interval(s) where $f(x)$ is decreasing.

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

g. Find $\lim_{x\to\infty} f(x) = 0$

h. As x increases without bound the $f(x)$…

approaches zero

i. Estimate the point of inflection.

$(-\frac{1}{2},-1)$

j. Find the average rate of change over the interval $-1 \le x \le 0$.

$f(-1)=0$
$f(0)=-2$

$\frac{-2-0}{0-(-1)} = \frac{-2}{1} = \boxed{-2}$

k. Find $\lim_{x\to-\infty} f(x) = 0$

l. As x decreases without bound the $f(x)$…

approaches zero

m. Find the horizontal asymptote.

$y=0$

n. State the interval(s) where $f(x) < 0$.

$(-\infty,-2)\cup(-1,2)$

13. Which one of the following polynomials could be the denominator for the graph of the rational function shown above in question #12 ?

(A) $x^2 + 4x + 4$

(B) $x^2 - 4x + 4$

(C) $x^2 - 4$

(D) $(x-4)^2$

(E) $(x+4)^2$

$x \neq -2 \quad x \neq 2$

$(x+2)(x-2) \neq 0$

$x^2 + 2x - 2x - 4 \neq 0$

$x^2 - 4 \neq 0$

1.7A Rational Functions and End Behavior

1.7A Test Prep

Multiple Choice

14. Given $f(x) = x^2 + a^2$ and $g(x) = x^2 - a^2$ where a is a constant integer. The function $r(x) = \frac{f(x)}{g(x)}$. What is the domain of $r(x)$?

$x^2 - a^2 \neq 0$

$(x+a)(x-a) \neq 0$

$x \neq -a \quad x \neq a$

$g \neq 0$

(A) $(-\infty, -a) \cup (a, \infty)$

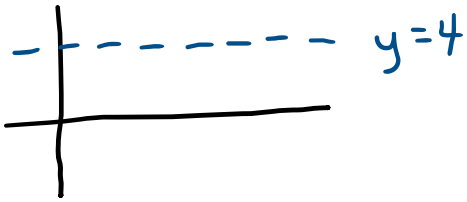
(B) $(-a, a)$

(C) $(-\infty, -a) \cup (-a, a) \cup (a, \infty)$

(D) $(-\infty, -a)$

(E) (a, ∞)

15. The function $f(x)$ is defined such that for all $x > 0$, the line $y = 4$ is a horizontal asymptote. Which of the following must be true?



(A) $f(4)$ is undefined maybe

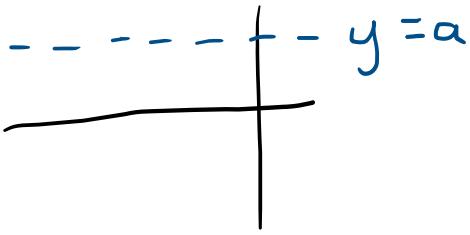
(B) $f(x) \neq 4$ for all $x > 0$ maybe

(C) $\lim_{x \to \infty} f(x) = 0$ nope

(D) $\lim_{x \to \infty} f(x) = 4$ yes!!

(E) $f(4) = 0$ maybe

16. Given the line $y = a$ is a horizontal asymptote of $g(x)$ for all $x < 0$, which of the following must be true?



(A) $g(a)$ is undefined maybe

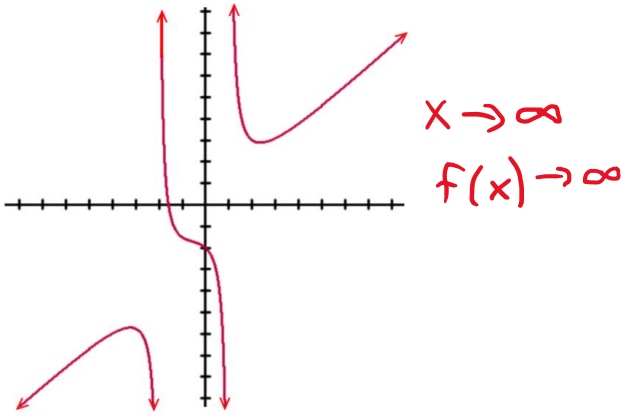
(B) $g(x) \neq a$ maybe

(C) $\lim_{x \to -\infty} g(x) = a$ yes!

(D) $\lim_{x \to -\infty} g(x) = -a$ nope

(E) None of the above nope

17. Given the graph of f. Which describes the end behavior of f?



(A) $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$

(B) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

(C) $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

(D) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$

(E) $\lim_{x \to \infty} f(x) = f(0)$