## Zeros

## Example 1:

$$
f(x)=\frac{x^{2}+2 x-8}{2 x^{2}+4}
$$

Domain:

Zero(s):


## Holes

## Example 2:

$$
g(x)=\frac{x^{2}-x-12}{x-4}
$$

Domain:

Hole(s):

Zero(s):


## Zeros

Let $f$ be the rational function $f(x)=\frac{N(x)}{D(x)}$ where $N$ and $D$ have no common factors. The zeros of the rational function occur when $N(x)=0$ for all $x$ in the domain of $f$.

## Vertical Asymptotes

## Example 3:

$$
f(x)=\frac{4 x+8}{x^{2}+3 x-10}
$$

Domain:

Hole(s):

Zero(s):

Vertical Asymptote(s):


## Example 4:

$$
h(x)=\frac{x^{2}-9}{x^{2}-2 x-3}
$$

Domain:
Hole(s):

Zero(s):

Vertical Asymptote(s):

Horizontal Asymptote:
$y$-intercept:

## Example 5:



Domain:

Hole(s):

Zero(s):

Vertical Asymptote(s):

Horizontal Asymptote:
$y$-intercept (estimate):

## Sign Table for Example \#5

| $\boldsymbol{x}$ | $-\infty<x<-4$ | -4 | $-4<x<-2$ | -2 | $-2<x<2$ | 2 | $2<x<7$ | 7 | $7<x<\infty$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ |  |  |  |  |  |  |  |  |  |

### 1.8 Rational Functions and Zeros

Find the zeros of the following rational function if one exists.

1. $f(x)=\frac{x-1}{x^{2}-9}$
2. $d(t)=\frac{(t+3)(t-1)}{4 t+12}$
3. $h(x)=\frac{x^{2}-3 x-10}{x^{2}+6 x}$
4. $r(x)=\frac{x-1}{x}$
5. 

| $\boldsymbol{x}$ | $-\infty<x<-3$ | -3 | $-3<x<5$ | 5 | $5<x<\infty$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | Positive | DNE | Negative | 0 | Positive |

6. $c(n)=\frac{n^{2}+5 n}{n^{2}-25}$
7. 

| $\boldsymbol{x}$ | $-\infty<x<1$ | 1 | $1<x<6$ | 6 | $6<x<\infty$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{g}(\boldsymbol{x})$ | Negative | 0 | Negative | 0 | Positive |

## Use the rational function to answer the following.

8. 

$$
f(x)=\frac{x^{2}-2 x-24}{x-6}
$$

a. Domain:
b. Hole(s):
c. Zero(s):
d. Vertical Asymptote(s):
e. Horizontal Asymptote:
f. $y$-intercept:
9.

$$
g(x)=\frac{4(x+5)(x-2)}{x^{2}-4}
$$

a. Domain:
b. Hole(s):
c. Zero(s):
d. Vertical Asymptote(s):
e. Horizontal Asymptote:
f. $y$-intercept:

## Use the rational function to answer the following.

10. 

$$
f(x)=\frac{x+2}{3 x^{2}+6 x}
$$

a. Domain:
b. Hole(s):
c. $x$-intercept(s):
d. Vertical Asymptote(s):
e. Horizontal Asymptote:
f. $y$-intercept:
11.

$$
h(t)=\frac{t^{3}-2 t^{2}}{t^{2}+3 t-18}
$$

a. Domain:
b. Hole(s):
c. $\operatorname{Root}(\mathrm{s})$ :
d. Vertical Asymptote(s):
e. Horizontal Asymptote:
f. $y$-intercept:

Use the graph to create a sign table.
12.


| $\boldsymbol{x}$ | $-\infty<x<-5$ | -5 | $-5<x<-2$ | -2 | $-2<x<5$ | 5 | $5<x<\infty$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ |  |  |  |  |  |  |  |

13. 


(C) The Algebros from FlippedMath.com

## Multiple Choice

14. The function $f$ is given by $f(x)=\frac{x^{2}+2 x-24}{4-x}$. Which of the following describes the function $f$ ?
(A) The graph of $f$ has an $x$-intercept at $x=-6$ and a vertical asymptote of $x=4$.
(B) The graph of $f$ has an $x$-intercept at $x=-6$ and a hole at $x=4$.
(C) The graph of $f$ has an $x$-intercept at $x=-6$ and a vertical asymptote of $x=-4$.
(D) The graph of $f$ has an $x$-intercept at $x=-6$ and a hole at $x=-4$.
(E) The graph of $f$ has $x$-intercepts at $x=-6$ and $x=4$.

For questions 15 and 16 use the following table.

| $\boldsymbol{x}$ | $-\infty<x<-3$ | -3 | $-3<x<0$ | 0 | $0<x<2$ | 2 | $2<x<\infty$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | positive | 0 | negative | undefined | negative | 0 | positive |

15. Which of the following must be true for the function $f$ ?
(A) The graph of $f$ has a maximum at $x=-3$ and a minimum at $x=2$.
(B) The graph of $f$ has a minimum at $x=-3$ and a maximum at $x=2$.
(C) $f$ has exactly two distinct real zeros.
(D) $f$ has exactly three distinct real zeros.
(E) The graph of $f$ has a vertical asymptote at $x=0$.
16. Which of the following could be an expression for $f(x)$ ?
(A) $\frac{x(x+3)(x-2)}{x}$
(B) $\frac{x(x-3)(x+2)}{x}$
(C) $\frac{x}{x(x+3)(x-2)}$
(D) $\frac{x}{x(x-3)(x+2)}$
(E) None of the above
