1.8 Rational Functions and Zeros

AP Precalculus

1.8 Practice

Find the zeros of the following rational function if one exists.

1. $f(x) = \frac{x-1}{x^2-9}$ $= 0$

$(x+3)(x-3)$

$x = 1$

2. $d(t) = \frac{(t+3)(t-1)}{4t+12}$ $= 0$

$4(t+3)$

$t = 1$

($t = -3$ is a hole so not a zero)

3. $h(x) = \frac{x^2-3x-10}{x^2+6x}$

$(x-5)(x+2) = 0$

$x(x+6)$

$x = 5$ and -2

4. $r(x) = \frac{x-1}{x}$ $= 0$

$x = 1$

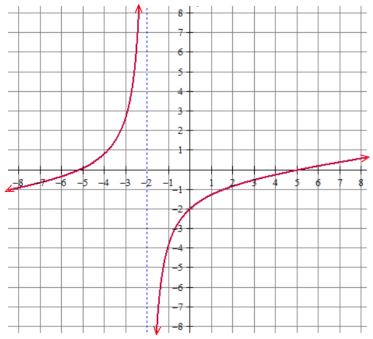
5.

x	$-\infty < x < -3$	-3	$-3 < x < 5$	5	$5 < x < \infty$
$f(x)$	Positive	DNE	Negative	0	Positive

$x = 5$

Use the graph to create a sign table.

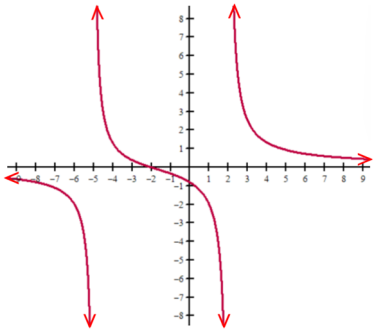
6.



x	$-\infty < x < -5$	-5	$-5 < x < -2$	-2	$-2 < x < 5$	5	$5 < x < \infty$
$f(x)$	negative	0	positive	DNE	negative	0	positive

Does Not Exist (undefined)

7.



x	$-\infty < x < -5$	-5	$-5 < x < -2$	-2	$-2 < x < 2$	2	$2 < x < \infty$
$h(x)$	negative	DNE	positive	0	negative	DNE	positive

Does Not Exist (undefined)

CALCULATOR ACTIVE - Find the zeros of the following rational function if one exists.

8. $f(x) = \frac{x^2+5x-2}{x^2-9}$

from graph

$x = -5.372$ and 0.372

9. $d(t) = \frac{t^2+5t-9}{4t+12}$

from graph

$x = -6.405$ and 1.405

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Use the rational function to answer the following.

10. $(x-6)(x+4)=0$

$$f(x)=\frac{x^2-2x-24}{x-6}\quad x-6\neq 0$$

a. Domain:

$(-\infty,6)\cup(6,\infty)$

b. Zero(s):

$x=-4$

($x=6$ is a hole so not a zero)

c. Horizontal Asymptote:

none

d. y-intercept: $(0,4)$

$f(0)=\frac{(0)^2-2(0)-24}{0-6}=\frac{-24}{-6}=4$

e. Where is $f(x)>0$? $(-4,6)\cup(6,\infty)$

	-5		-4		0		6		7
x	$(-\infty,-4)$		-4		$(-4,6)$		6		$(6,\infty)$
f(x)	$\pm$	=	0	=	=		DNE		$\pm$
	neg				pos				pos

11. $4x^2+12x-40$

$4(x^2+3x-10)$

$$g(x)=\frac{4(x+5)(x-2)}{x^2-4}=0$$

$(x+2)(x-2)\neq 0$

a. Domain:

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

b. Zero(s):

$x=-5$

($x=2$ is a hole so not a zero)

c. Horizontal Asymptote:

$y=\frac{4}{1}=4$

d. y-intercept: $(0,10)$

$g(0)=\frac{4(0)^2+12(0)-40}{(0)^2-4}=\frac{-40}{-4}=10$

e. Where is $f(x)<0$? $(-5,-2)$

	-6		-5		-3		-2		0		3
x	$(-\infty,-5)$		-5		$(-5,-2)$		-2		$(-2,3)$		$(3,\infty)$
f(x)	$\pm$	=	0	=	=		DNE		=		$\pm$
	pos				neg				pos		pos

12.

$$f(x)=\frac{x+3}{3x^2+6x}=0$$

$3x(x+2)\neq 0$

a. Domain:

$(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

b. Zero(s):

$x=-3$

c. Horizontal Asymptote:

$y=0$

d. y-intercept: *DNE*

$f(0)=\frac{0+3}{3(0)^2+6(0)}=\frac{3}{0}=$ *does not exist*

e. Where is $f(x)<0$? $(-\infty,-3)\cup(-2,0)$

	-4		-3		-2.5		-2		-1		0		1
x	$(-\infty,-3)$		-3		$(-3,-2)$		-2		$(-2,0)$		0		$(0,\infty)$
f(x)	=	=	0	=	$\pm$	=	DNE	=	$\pm$	=	DNE		$\pm$
	neg				pos				neg				pos

13. $t^2(t-2)=0$

$$h(t)=\frac{t^3-2t^2}{t^2+3t-18}$$

$(t+6)(t-3)\neq 0$

a. Domain:

$(-\infty,-6)\cup(-6,3)\cup(3,\infty)$

b. Zero(s):

$t=0$ and 2

c. Horizontal Asymptote:

none

d. y-intercept: $(0,0)$

$h(0)=\frac{0}{-18}=0$

e. Where is $f(x)>0$? $(-6,0)\cup(0,2)\cup(3,\infty)$

	-7		-6		-1		0		1		2.5		3		4
x	$(-\infty,-6)$		-6		$(-6,0)$		0		$(0,3)$		2		$(2,3)$		$(3,\infty)$
f(x)	=	=	DNE	=	=	=	0	=	=	=	0	=	$\pm$	=	$\pm$
	neg				pos				pos				neg		pos

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Multiple Choice

zero → $(x+6)(x-4)=0$ ← hole

14. The function f is given by $f(x)=\frac{x^2+2x-24}{4-x}$. Which of the following describes the function f ?

$-(x-4)$

(A) The graph of f has an x-intercept at $x=-6$ and a vertical asymptote of $x=4$.

(B) The graph of f has an x-intercept at $x=-6$ and a hole at $x=4$.

(C) The graph of f has an x-intercept at $x=-6$ and a vertical asymptote of $x=-4$.

(D) The graph of f has an x-intercept at $x=-6$ and a hole at $x=-4$.

(E) The graph of f has x-intercepts at $x=-6$ and $x=4$.

For questions 15 and 16 use the following table.

zero zero

x	$-\infty<x<-3$	-3	$-3<x<0$	0	$0<x<2$	2	$2<x<\infty$
$f(x)$	positive	0	negative	undefined	negative	0	positive

hole or vertical asymptote

15. Which of the following must be true for the function f ?

(A) The graph of f has a maximum at $x=-3$ and a minimum at $x=2$.

(B) The graph of f has a minimum at $x=-3$ and a maximum at $x=2$.

(C) f has exactly two distinct real zeros. $x=-3$ and $x=2$

(D) f has exactly three distinct real zeros.

(E) The graph of f has a vertical asymptote at $x=0$.

16. Which of the following could be an expression for $f(x)$?

(A) $\frac{x(x+3)(x-2)}{x}=0$

(B) $\frac{x(x-3)(x+2)}{x}$

(C) $\frac{x}{x(x+3)(x-2)}$

(D) $\frac{x}{x(x-3)(x+2)}$

(E) None of the above

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$(x+2)(x-2)$ (−2, 2)

17. The function r is given by $r(x) = \frac{x^2-4}{x+3}$. On what intervals of x is $r(x) \geq 0$?

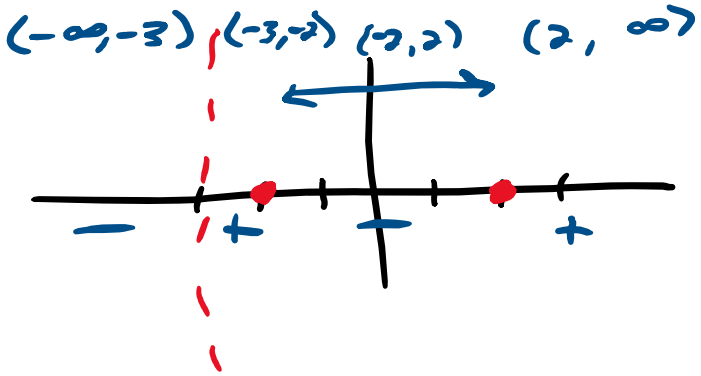
(A) $x \geq 2$ only

(B) $-2 \leq x \leq 2$ only

(C) $-3 < x \leq -2$ and $x \geq 2$

(D) $x < -3$ only

−3



x	$(-\infty, -3)$	-3	$(-3, -2)$	-2	$(-2, 2)$	2	$(2, \infty)$
$r(x)$	$\frac{+}{-}$ negative	DNE	$\frac{+}{+}$ positive	0	$\frac{-}{+}$ negative	0	$\frac{+}{+}$ positive

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$x(x-4)$ (0, 4)

18. The function r is given by $r(x) = \frac{x^2-4x}{x^2+8x+16}$. On what intervals of x is $r(x) < 0$?

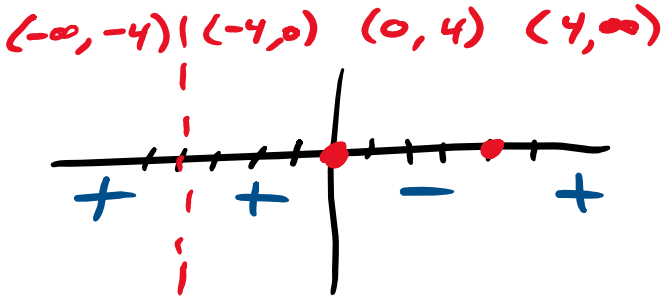
$(x+4)(x+4)$ -4 -4

(A) $x > 4$

(B) $0 < x < 4$

(C) $-4 < x < 4$

(D) $x < -4$



x	$(-\infty, -4)$	-4	$(-4, 0)$	0	$(0, 4)$	4	$(4, \infty)$
$r(x)$	$\frac{+}{+}$ positive	DNE	$\frac{+}{+}$ positive	0	$\frac{-}{+}$ negative	0	$\frac{+}{+}$ positive

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