1.8 Rational Functions and Zeros

AP Precalculus

1.8 Practice

Find the zeros of the following rational function if one exists.

1. $f(x) = \frac{x-1}{x^2-9}$ $=0$

$(x+3)(x-3)$

$x = 1$

2. $d(t) = \frac{\cancel{(t+3)}(t-1)}{4t+12}$ $=0$

$4\cancel{(t+3)}$

$t = 1$

3. $(x-5)(x+2)=0$

$h(x) = \frac{x^2-3x-10}{x^2+6x}$

$x(x+6)$

$x = 5 \text{ and } -2$

4. $r(x) = \frac{x-1}{x}$ $=0$

$x = 1$

5.

x	$-\infty < x < -3$	-3	$-3 < x < 5$	5	$5 < x < \infty$
$f(x)$	Positive	DNE	Negative	0	Positive

$x = 5$

6. $n\cancel{(n+5)} = 0$

$c(n) = \frac{n^2+5n}{n^2-25}$

$\cancel{(n+5)}(n-5)$

$n = 0$

7.

x	$-\infty < x < 1$	1	$1 < x < 6$	6	$6 < x < \infty$
$g(x)$	Negative	0	Negative	0	Positive

$x = 1 \text{ and } 6$

Use the rational function to answer the following.

8. $\cancel{(x-6)}(x+4) = 0$

$$f(x) = \frac{x^2-2x-24}{\cancel{x-6}} \neq 0$$

a. Domain:

$(-\infty, 6) \cup (6, \infty)$

b. Hole(s):

$x = 6$

c. Zero(s):

$x = -4$

d. Vertical Asymptote(s):

none

e. Horizontal Asymptote:

none

f. y-intercept:

$f(0) = \frac{-24}{-6} = 4$

9. $4x^2 + 12x - 40$

$4(x^2 + 3x - 10)$

$$g(x) = \frac{4(x+5)\cancel{(x-2)}}{x^2-4} = 0$$

$(x+2)\cancel{(x-2)} \neq 0$

a. Domain:

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

b. Hole(s):

$x = 2$

c. Zero(s):

$x = -5$

d. Vertical Asymptote(s):

$x = -2$

e. Horizontal Asymptote:

$y = \frac{4}{1} = 4$

f. y-intercept:

$g(0) = \frac{-40}{-4} = 10$

Use the rational function to answer the following.

10.

$$f(x) = \frac{\cancel{x+2}}{3x^2+6x}$$ $= 0$

$3x(\cancel{x+2}) \neq 0$

g. Domain:

$(-\infty,-2) \cup (-2,0) \cup (0,\infty)$

h. Hole(s):

$x = -2$

i. Zero(s):

none

j. Vertical Asymptote(s):

$x = 0$

k. Horizontal Asymptote:

$y = 0$

l. *y*-intercept:

$f(0) = \frac{2}{0} = \textit{does not exist}$

11.

$t^2(t-2) = 0$

$$h(t) = \frac{t^3 - 2t^2}{t^2 + 3t - 18}$$

$(t+6)(t-3) \neq 0$

a. Domain:

$(-\infty,-6) \cup (-6,3) \cup (3,\infty)$

b. Hole(s):

none

c. Zero(s):

$x = 0$ and 2

d. Vertical Asymptote(s):

$x = -6$ and 3

e. Horizontal Asymptote:

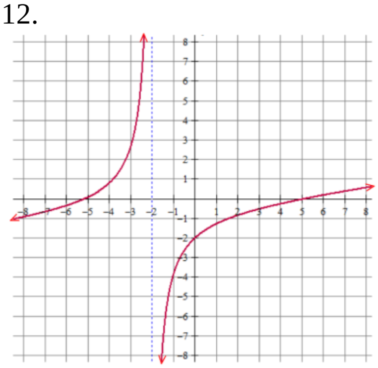
none

f. *y*-intercept:

$h(0) = \frac{0}{-18} = 0$

Use the graph to create a sign table.

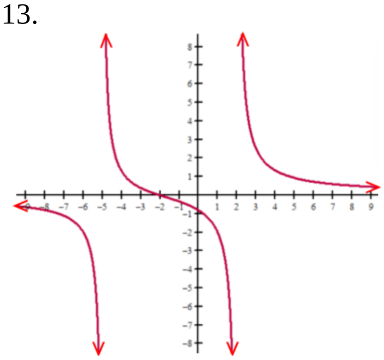
12.



x	$-\infty < x < -5$	-5	$-5 < x < -2$	-2	$-2 < x < 5$	5	$5 < x < \infty$
$f(x)$	negative	0	positive	DNE	negative	0	positive

Does Not Exist (undefined)

13.



x	$-\infty < x < -5$	-5	$-5 < x < -2$	-2	$-2 < x < 2$	2	$2 < x < \infty$
$h(x)$	negative	DNE	positive	0	negative	DNE	positive

Does Not Exist (undefined)

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Multiple Choice

zero hole

$(x+6)\cancel{(x-4)} = 0$

14. The function f is given by $f(x) = \frac{x^2+2x-24}{4-x}$. Which of the following describes the function f ?

$-\cancel{(x-4)}$

(A) The graph of f has an x-intercept at $x = -6$ and a vertical asymptote of $x = 4$.

(B) The graph of f has an x-intercept at $x = -6$ and a hole at $x = 4$.

(C) The graph of f has an x-intercept at $x = -6$ and a vertical asymptote of $x = -4$.

(D) The graph of f has an x-intercept at $x = -6$ and a hole at $x = -4$.

(E) The graph of f has x-intercepts at $x = -6$ and $x = 4$.

For questions 15 and 16 use the following table.

zero zero

x	$-\infty < x < -3$	-3	$-3 < x < 0$	0	$0 < x < 2$	2	$2 < x < \infty$
$f(x)$	positive	0	negative	undefined	negative	0	positive

hole or vertical asymptote

15. Which of the following must be true for the function f ?

(A) The graph of f has a maximum at $x = -3$ and a minimum at $x = 2$.

(B) The graph of f has a minimum at $x = -3$ and a maximum at $x = 2$.

(C) f has exactly two distinct real zeros. $x = -3$ and $x = 2$

(D) f has exactly three distinct real zeros.

(E) The graph of f has a vertical asymptote at $x = 0$.

16. Which of the following could be an expression for $f(x)$?

(A) $\frac{\cancel{x}(x+3)(x-2)}{\cancel{x}} = 0$

(B) $\frac{x(x-3)(x+2)}{x}$

(C) $\frac{x}{x(x+3)(x-2)}$

(D) $\frac{x}{x(x-3)(x+2)}$

(E) None of the above