1.9 Rational Functions and Vertical Asymptotes

AP Precalculus

1.9 Practice

Find the domain and vertical asymptote(s) of the following rational function if one exists.

1. $f(x) = \frac{x(x+2)}{x^2-4}$

$(x+2)(x-2)$

Domain:

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Vertical Asymptote(s):

$x = 2$

2. $d(t) = \frac{t^2+4t-12}{(t-2)^2}$

$(t+6)(t-2)$

$(t-2)(t-2)$

Domain:

$(-\infty, 2) \cup (2, \infty)$

Vertical Asymptote(s):

$x = 2$

3. $h(x) = \frac{x^3-3x^2}{x^2+8x+15}$

$x^2(x-3)$

$(x+3)(x+5)$

Domain:

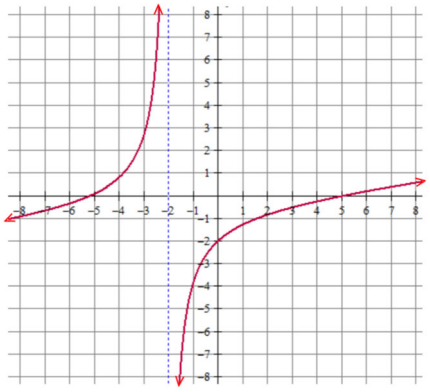
$(-\infty, -5) \cup (-5, -3) \cup (-3, \infty)$

Vertical Asymptote(s):

$x = -3$ and -5

State the vertical asymptotes(s) of the following rational functions. Use limit notation.

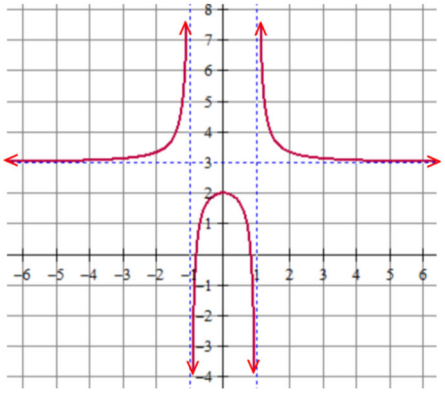
4.



Limit Notation Vertical Asymptote(s):

$\lim_{x \to -2^-} f(x) = \infty$ $\lim_{x \to -2^+} f(x) = -\infty$

5.

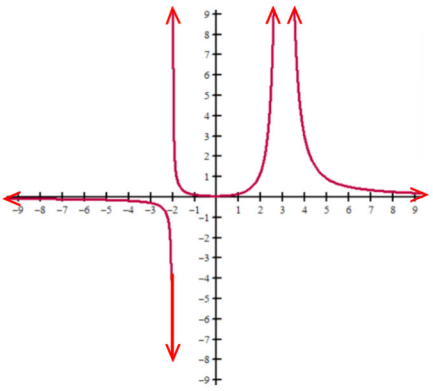


Limit Notation Vertical Asymptote(s):

$\lim_{x \to -1^-} f(x) = \infty$ $\lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to 1^-} f(x) = -\infty$ $\lim_{x \to 1^+} f(x) = \infty$

6.



Limit Notation Vertical Asymptote(s):

$\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = \infty$

$\lim_{x \to 3^-} f(x) = \infty$ $\lim_{x \to 3^+} f(x) = \infty$

CALCULATOR ACTIVE Complete the table to answer the following.

7. $f(x) = \frac{x^2-1}{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	-142.1	-1492	-14992	*undefined*	15008	1508	158.1

Vertical Asymptote:

$x = 4$

Limit Notation of Vertical Asymptote:

$\lim_{x \to 4^-} f(x) = -\infty$ $\lim_{x \to 4^+} f(x) = \infty$

CALCULATOR ACTIVE Complete the table to answer the following.

8. $f(x) = \frac{x^2-2x}{x+2}$

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$	-86.1	-806	-8006	$undefinded$	7994	794	74.1

Vertical Asymptote:

$x = -2$

Limit Notation of Vertical Asymptote:

$\lim_{x\to -2^-} f(x) = -\infty$ $\lim_{x\to -2^+} f(x) = \infty$

Use the table of the rational function *h* to find the following.

9.

t	$d(t)$
-0.1	5,589
-0.01	37,231
-0.001	96,543
-0.0001	148,234
0	undefined
0.0001	128,341
0.001	89,437
0.01	18,235
0.1	1,455

a. Find $d(0) =$ undefined

b. Find the y-intercept.
Does not exist!

c. Find $\lim_{t\to 0^-} d(t) = \infty$

d. Find $\lim_{t\to 0^+} d(t) = \infty$

e. As t approaches zero from the left the $d(t)$…
approaches infinity

f. As t approaches zero from the right the $d(t)$…
approaches infinity

Make a sketch of the rational function with the following characteristics.

10. The graph of f has…

a. $f(-4) = 0$

b. $f(6) = 0$

c. $\lim_{x\to -3^-} f(x) = -\infty$

d. $\lim_{x\to -3^+} f(x) = \infty$

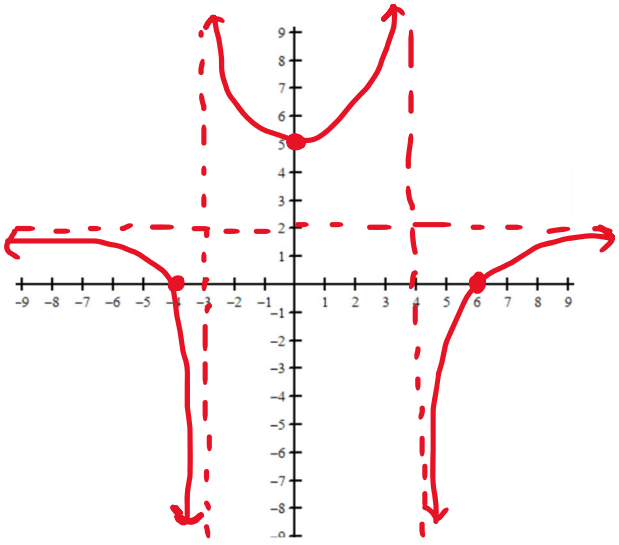
e. $\lim_{x\to 4^-} f(x) = \infty$

f. $\lim_{x\to 4^+} f(x) = -\infty$

g. $\lim_{x\to -\infty} f(x) = 2$

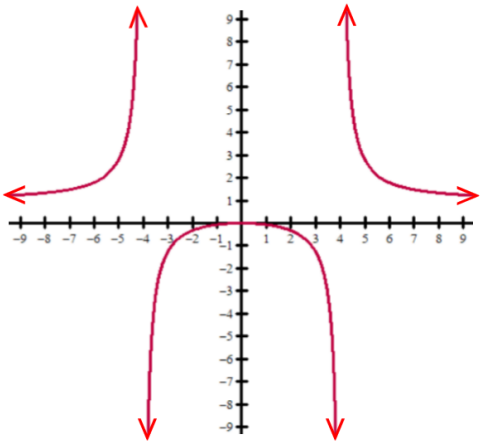
h. $\lim_{x\to \infty} f(x) = 2$

i. $f(0) = 5$



Multiple Choice

11. Given the graph of f. Which of the following describes the function f?



(A) $\lim_{x\to -4^-} f(x) = -\infty$ and $\lim_{x\to -4^+} f(x) = -\infty$

(B) $\lim_{x\to -4^-} f(x) = \infty$ and $\lim_{x\to -4^+} f(x) = -\infty$

(C) $\lim_{x\to -4^-} f(x) = -\infty$ and $\lim_{x\to -4^+} f(x) = \infty$

(D) $\lim_{x\to -4^-} f(x) = \infty$ and $\lim_{x\to -4^+} f(x) = \infty$

(E) $\lim_{x\to -4} f(x) = f(0)$

Free Response

12. The function f is a rational function graphed in the xy-plane. The polynomial in the numerator of f has exactly one real zero at $x = 3$. The polynomial of the denominator of f has exactly two real zeros at both $x = 3$ and $x = 6$. The multiplicities of the zeros at $x = 3$ in the numerator and in the denominator are equal.

a. Find the domain for the graph of f.

$(-\infty, 3) \cup (3,6) \cup (6, \infty)$

b. Find any holes and vertical asymptotes for the graph of f if they exist. Explain what causes the holes or vertical asymptotes.

$x = 3$ is a hole because the polynomial in the numerator and the polynomial in the denominator share a common factor $(x - 3)$.

$x = 6$ is a vertical asymptote because of the zero in the denominator.

c. Explain how your answer from part b would change if the multiplicities of the zeros at $x = 3$ in the numerator and denominator were not equal?

If the multiplicity of the zero in the numerator was greater than the denominator, then there would still be a hole at $x = 3$. The overall graph might look different, but there is still a hole.

If the multiplicity of the zero in the denominator was greater than the numerator, then there would not be a hole at $x = 3$, instead there would be a vertical asymptote at $x = 3$.