

2.1 Change in Arithmetic and Geometric Sequences

AP Precalculus

Solutions

2.1 Practice

Find an equation that gives the n th term of each sequence. Use the initial value ($k = 0$) of the sequence in your equation.

1. $\{9, 27, 81, 243, \dots\}$

$\times 3$ $\times 3$ $\times 3$

$r = 3$
 $g_0 = 9 \div 3 = 3$

$g_n = 3(3)^n$

2. $\{-4, -1, 2, 5, \dots\}$

$+3$ $+3$ $+3$

$d = 3$

$a_0 = -4 - 3 = -7$

$a_n = -7 + 3n$

3. $\{-1, -4, -16, -64, \dots\}$

$\times 4$ $\times 4$ $\times 4$

$r = 4$

$g_0 = -1 \div 4 = -\frac{1}{4}$

$g_n = -\frac{1}{4}(4)^n$

Find an equation that gives the n th term of each sequence. Use the initial value ($k = 0$) of the sequence in your equation.

4. $\{10, 5, \frac{5}{2}, \frac{5}{4}, \dots\}$
 $\times \frac{1}{2} \quad \times \frac{1}{2} \quad \times \frac{1}{2}$
 $r = \frac{1}{2}$
 $a_0 = 10 \times 2 = 20$

$$a_n = 20 \left(\frac{1}{2}\right)^n$$

5. $\{-5, -9, -13, -17, \dots\}$
 $-4 \quad -4 \quad -4$
 $d = -4$
 $a_0 = -5 + 4 = -1$

$$a_n = -1 - 4n$$

6. $\{\frac{4}{3}, 1, \frac{3}{4}, \frac{9}{16}, \dots\}$
 $? \quad ? \quad ?$
 Not sure about r ?
 Divide a term by the previous term.

$$r = \frac{3}{4} \div 1 = \frac{3}{4}$$

$$a_0 = \frac{4}{3} \div \frac{3}{4} = \frac{4}{3} \cdot \frac{4}{3} = \frac{16}{9}$$

$$a_n = \frac{16}{9} \left(\frac{3}{4}\right)^n$$

Find an equation that gives the n th term of each sequence. Instead of the initial value use the k th term of the sequence in your equation. k is given for each problem.

7. $\{75, 15, 3, \frac{3}{5}, \dots\} \quad k = 2$
 $\times \frac{1}{5} \quad \times \frac{1}{5} \quad \times \frac{1}{5}$
 $r = \frac{1}{5} \quad a_2 = 15$

$$a_n = 15 \left(\frac{1}{5}\right)^{n-2}$$

8. $\{44, 36, 28, 20, \dots\} \quad k = 4$
 $-8 \quad -8 \quad -8$
 $d = -8 \quad a_4 = 20$

$$a_n = 20 - 8(n-4)$$

9. $\{-10, -100, -1,000, -10,000, \dots\}$
 $k = 3 \quad \times 10 \quad \times 10 \quad \times 10$
 $r = 10 \quad a_3 = -1,000$

$$a_n = -1,000 (10)^{n-3}$$

10. $\{11, 22, 33, 44, \dots\} \quad k = 2$
 $+11 \quad +11 \quad +11$
 $d = 11 \quad a_2 = 22$

$$a_n = 22 + 11(n-2)$$

11. $\{\frac{9}{4}, \frac{3}{2}, 1, \frac{2}{3}, \dots\} \quad k = 3$
 $\times \frac{2}{3} \quad \times \frac{2}{3}$
 $r = \frac{2}{3} \quad a_3 = 1$

$$a_n = \left(\frac{2}{3}\right)^{n-3}$$

12. $\{\frac{1}{9}, \frac{1}{3}, 1, 3, \dots\} \quad k = 2$
 $\times 3 \quad \times 3 \quad \times 3$
 $r = 3 \quad a_2 = \frac{1}{3}$

$$a_n = \frac{1}{3} (3)^{n-2}$$

Find the n th term of each sequence. Write an equation for each sequence before finding the n th term.

13. $\{105, 95, 85, 75, \dots\}$ What is the 87th term?

$$d = -10 \quad a_1 = 105$$

$$a_n = 105 - 10(n-1)$$

$$a_{87} = 105 - 10(87-1) = -755$$

There are many different equations you could use, but the 87th term will be the same answer.

14. $\{\frac{1}{6}, 1, \frac{11}{6}, \frac{8}{3}, \dots\}$ What is the 34th term?

$$d = \frac{5}{6} \quad a_2 = 1$$

$$a_n = 1 + \frac{5}{6}(n-2)$$

$$a_{34} = 1 + \frac{5}{6}(32) = \frac{83}{3} \text{ or } 27.6666$$

15. $\{-86, -80, -74, -68, \dots\}$ What is the 52nd term?

$$d = 6 \quad a_1 = -86$$

$$a_n = -86 + 6(n-1)$$

$$a_{52} = -86 + 6(51) = 220$$

16. $\{-21, -25, -29, -33, \dots\}$ What is the 120th term?

$$d = -4 \quad a_1 = -21$$

$$a_n = -21 - 4(n-1)$$

$$a_{120} = -21 - 4(119) = -497$$

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2.1 Test Prep

17. Which of the following equations could represent the sequence $\{10, 2, -6, -14, \dots\}$?

$$a_0 = 18 \quad d = -8$$

✓ I. $f(n) = 10 - 8(n - 1)$

✗ II. $f(n) = -8 + 10n$

✗ III. $f(n) = -8 + 10(n - 1)$

✓ IV. $f(n) = 18 - 8n$

✗ V. $f(n) = 10 - 8n$

✗ VI. $f(n) = 2 - 8(n + 2)$

I and IV

18. Which of the following equations could represent the sequence $\{\frac{1}{2}, 2, 8, 32, \dots\}$?

$$r = 4 \quad q_0 = \frac{1}{8}$$

✗ I. $f(n) = \frac{1}{2} \left(\frac{1}{4}\right)^{n-1}$ $r = 4$

✓ II. $f(n) = \frac{1}{2}(4)^{n-1}$

✗ III. $f(n) = 8(4)^{n+3}$

✗ IV. $f(n) = 2 \left(\frac{1}{4}\right)^{n-2}$

✓ V. $f(n) = \frac{1}{8}(4)^n$

II and V

19. The first term of an arithmetic sequence is 4, and the common difference of the sequence is 3. What is the seventh term of the sequence?

(A) 22

(B) 25

(C) 972

(D) 2,916

A

$$a_n = 4 + 3(n - 1)$$

$$a_7 = 4 + 3(7 - 1) = 22$$