### 2.2 Change in Linear and Exponential Functions

## Linear Functions vs. Arithmetic Sequences

## Arithmetic Sequence

$$
a_{n}=a_{0}+d n
$$

$$
\text { Initial Value }=
$$

$$
\text { Constant rate of change }=
$$

When not starting with the initial value, we can transform the above equations by using the $k$ th term. Our arithmetic sequence is...

Point-slope form of a linear function

$$
f(x)=\quad \quad a_{n}=a_{k}+d(n-k)
$$

Arithmetic Sequence with a known $\boldsymbol{k}$ th term

The point $\left(x_{1}, y_{1}\right)$ for a linear function is similar to the term $\left(k, a_{k}\right)$ of an arithmetic sequence.

## Exponential Functions vs. Geometric Sequences

| Exponential Function | Geometric Sequence |
| :---: | :---: |
| $f(x)=$ | $g_{n}=g_{0} r^{n}$ |

When not starting with the initial value, we can transform the above equations by using the $k$ th term. Our geometric sequence is

| Shifted Exponential Function | Geometric Sequence with a known $k$ th term |
| :---: | :---: |
| $f(x)=$ | $g_{n}=g_{\boldsymbol{k}} r^{(n-k)}$ |

The point $\left(x_{1}, y_{1}\right)$ for an exponential function is similar to the term $\left(k, g_{k}\right)$ of a geometric sequence.

## Domain

Remember the domain of a sequence won't be the same as the domain of its corresponding function.


The domain of this sequence is $\qquad$


The domain of this function is $\qquad$

Linear functions and exponential functions can both be expressed analytically in terms of an initial value and a constant involved with change. There is a difference between the two though. Linear functions are based on addition. Exponential functions are based on multiplication.

## Recognizing how to change the output values

Over equal-length input-value intervals, if the output values of a function change...
$\qquad$ , then the function is linear. (ADDING the SLOPE)
$\qquad$ , then the function is exponential. (MULTIPLYING the RATIO)

A function has the following coordinate points. Could the function represent a linear function, exponential function, or neither?

1. $(2,1),(3,4),(4,16)$
2. $(5,3),(6,7),(7,10)$
3. $(3,-1),(5,7),(7,15)$

If we know a function is linear or exponential (for sequences that would be arithmetic or geometric), then you only need two distinct values to come up with an equation (rule) for the function or sequence.
4. It is known that $f(x)$ is a linear function and that it passes through the points $(3,7)$ and $(8,1)$. Write an equation for this function.
5. It is known that $f(x)$ is an exponential function and that it passes through the points $(2,5)$ and $(4,12)$. Write an equation for this function.

The following functions are either linear or exponential. Identify the constant (slope or ratio) that causes the output values to change?
6. $y=4 \cdot 2^{x}$
7. $y=5 x+7$

## The following functions are either linear or exponential. Which is it? Justify your answer.

8. 

| $\boldsymbol{x}$ | 3 | 7 | 11 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 10 | 6 | 2 |

9. 

| $\boldsymbol{x}$ | 1 | 3 | 5 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 2 | 6 | 18 |

### 2.2 Change in Linear and Exponential Functions

A function has the following coordinate points. Could the function represent a linear function, exponential function, or neither?

1. $(1,2),(2,8),(3,20) \quad 2$.
$(10,30),(11,20),(12,10)$
2. $(13,20),(14,4),\left(15, \frac{4}{5}\right)$
3. $(7,6),(8,12),(9,20)$

The following functions are either linear or exponential. Which is it? Justify your answer.
5.

| $\boldsymbol{x}$ | 1 | 5 | 9 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 2 | 6 | 18 |

7. 

| $\boldsymbol{x}$ | 7 | 10 | 13 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 5 | 105 | 205 |

6. 

| $\boldsymbol{x}$ | -4 | -2 | 0 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 5 | 1 | -3 |

8. 

| $\boldsymbol{x}$ | 11 | 20 | 29 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 8 | 4 | 2 |

Is each function linear or exponential. Identify the constant (slope or ratio) that causes the output values to change?

| 9. $y=-\frac{1}{4} \cdot 7^{x}$ | $10 . y=4 x-6$ | $11 \cdot y=2-9 x$ | 12. $y=\frac{1}{3} \cdot\left(\frac{2}{5}\right)^{x}$ |
| :--- | :--- | :--- | :--- |
| $13 . y-7=-5(x+4)$ | $14 . y+1=\left(\frac{3}{4}\right)^{x+7}$ | $15 \cdot y+5=\frac{1}{6} \cdot 2^{x-3}$ | $16 \cdot y-6=3(x-10)$ |

It is known that $f(x)$ is a linear function and that it passes through the given points. Write an equation for this function.
17. $(1,4)$ and $(3,10)$
18. $(4,15)$ and $(10,3)$
19. $(10,2)$ and $(13,5)$

It is known that $f(x)$ is an exponential function and that it passes through the given points. Write an equation for this function.
20. $(1,4)$ and $(3,10)$
21. $(4,15)$ and $(10,3)$
22. $(10,2)$ and $(13,5)$

### 2.2 Change in Linear and Exponential Functions

### 2.2 Test Prep

23. Mr. Brust has been collecting He-Man figures for 40 years. The number of figures he owns can be modeled by an arithmetic sequence, where the first year is year 1 . The number of figures in year 5 was 12 , and the number of figures in year 20 was 167 . How many He-Man figures did he have in year 14?
24. Calculator active. Mr. Bean has lots of siblings and lots of nephews and nieces. The number of people in his family can be modeled using a geometric sequence, where the first generation is generation 1 . The number of people after generation 2 is 52 . The number of people after generation 4 is 468 . How many people will be in the Bean family after generation 6 ?
25. The third term of a sequence is 5 , and the fifth term of the sequence is 20 . Of the following, which statement is true?
(A) If the sequence is arithmetic, the first term could be -25 .
(B) If the sequence is arithmetic, the fourth term could be 7.5.
(C) If the sequence is geometric, the fourth term could be 7 .
(D) If the sequence is geometric, the sixth term could be 40 .
