## General Form of an Exponential Function

An exponential function has the general form

$$
f(x)=
$$

Where the initial value is $\qquad$ . Also, $\qquad$ and $\qquad$ .

when $a>0$ and $b>1$.

Exponential decay

when $a>0$ and $0<b<1$.

Identify if the exponential function is growth or decay and justify your response.

1. $f(x)=2(0.6)^{x}$

Exponential Growth or Decay
2. $f(x)=2\left(\frac{7}{5}\right)^{x}$

Exponential
Growth or Decay
3. $f(x)=\frac{4}{3}\left(\frac{1}{6}\right)^{x}$

Exponential
Growth or Decay
4. $f(x)=0.2(1.3)^{x}$

Exponential
Growth or Decay

Examine the function $f(x)=3(2)^{x}$. What are we doing to the initial value 3 when the input value is $x=5$ ?

$$
\begin{gathered}
f(5)=3(2)^{5} \\
f(x)=3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2
\end{gathered}
$$

The following values are output values of an exponential function of the form $f(x)=a \cdot b^{x}$, where $a$ and $b$ are constants. Write the function along with the input value that represents the output value.
5. $4 \cdot 4 \cdot 4 \cdot 1.5$
$f(x)=\quad$ where $x=$
6. $0.8 \cdot 3.2 \cdot 0.8$

$$
f(x)=\quad \text { where } x=
$$

Using your knowledge of exponential functions, sketch the general shape of each exponential function with the given parameters.


## Characteristics of Exponential Functions

Exponential Functions have the following characteristics.

- They are always increasing or always decreasing.
- There are no extrema. (Except on a closed interval.)
- Their graphs are always concave up or always concave down.
- Their graphs do not have inflection points.
- If the input values increase or decrease without bound, the end-behavior can be expressed as

$$
\lim _{x \rightarrow \pm \infty} a b^{x}=\quad \text { or } \quad \lim _{x \rightarrow \pm \infty} a b^{x}=\quad \text { or } \lim _{x \rightarrow \pm \infty} a b^{x}=
$$

## Answer the questions for each exponential function.

7. $f(x)=5(0.9)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$
8. $f(x)=-6(2)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$

## Additive Transformation of an Exponential Function

Let $g(x)=f(x)+k$. This just means we take a function $f$ and add something to it to shift the graph up or down. The function $f$ is having $k$ added to it to create $g$.

If the output values of $g$ are proportional over equal-length input-value intervals, then $f$ is exponential.

### 2.3 Exponential Functions

4. $f(x)=8\left(\frac{11}{5}\right)^{x}$

Exponential
Growth or Decay
8. $f(x)=18(5.6)^{x}$

Exponential
Growth or Decay

The following values are output values of an exponential function of the form $f(x)=a \cdot b^{x}$, where $a$ and $b$ are constants. Write the function along with the input value that represents the output value.
9. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7$
$f(x)=$
where $x=$
10. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 6$
$f(x)=$
where $x=$
11. $5 \cdot 5 \cdot 5$
$f(x)=$
where $x=$
12. $(-2) \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
$f(x)=$
where $x=$

## Answer the questions for each exponential function.

13. $f(x)=7(2)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$
14. $f(x)=(0.2)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$
15. $f(x)=-4(5)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$
16. $f(x)=-6(0.8)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$
17. $f(x)=6\left(\frac{1}{9}\right)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$
18. $f(x)=-(0.4)^{x}$
a. Is the function increasing or decreasing?
b. Is the function concave up or concave down?
c. Find $\lim _{x \rightarrow-\infty} f(x)=$
d. Find $\lim _{x \rightarrow \infty} f(x)=$

### 2.3 Exponential Functions

### 2.3 Test Prep

19. 

| $\boldsymbol{x}$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 40 | 20 | 10 | 5 | $\frac{5}{2}$ |

The exponential function $f$ is defined by $f(x)=a b^{x}$, where $a$ and $b$ are positive constants. The table gives values of $f(x)$ at selected values of $x$. Which of the following statements is true?
(A) $f$ demonstrates exponential decay because $a>0$ and $0<b<1$.
(B) $f$ demonstrates exponential decay because $a>0$ and $b>1$.
(C) $f$ demonstrates exponential growth because $a>0$ and $0<b<1$.
(D) $f$ demonstrates exponential growth because $a>0$ and $b>1$.
20. The function $h$ is a function of the form $h(x)=a \cdot b^{x}$, where $a \neq 0$ and $b>1$. The function $h$ is also given by $h(x)=f(x)+2$. Which of the following statements is true.
(A) The output values of both $f$ and $h$ are proportional over equal-length input-value intervals.
(B) The output values of $f$ only, not $h$, are proportional over equal-length input-value intervals.
(C) The output values of $h$ only, not $f$, are proportional over equal-length input-value intervals.
(D) The output values of neither $f$ nor $h$ are proportional over equal-length input-value intervals.

