2.4 Exponential Function Manipulation

Write your questions and thoughts here!

Recall: The **Negative Exponent Property** states that $b^{-n} =$

Recall: The **Product Property** for exponents: $b^m b^n =$

$$2^2 \cdot 2^3 = 2$$

Let's think of a **horizontal translation** of an exponential function. Remember, horizontal means we are doing something the x-values. $f(x) = b^{(x+k)}$, where b is a constant.

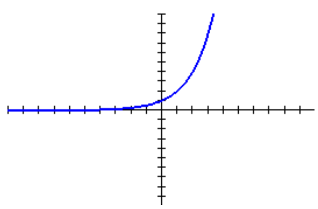
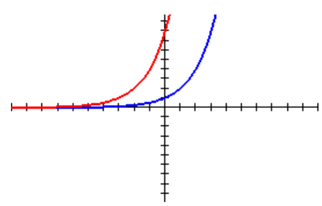
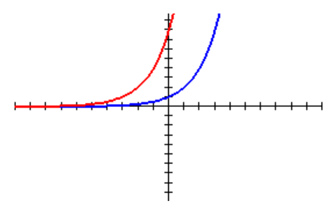
This now becomes a vertical dilation!

Horizontal Translation

Every horizontal translation of an exponential function, $f(x) = b^{(x+k)}$, is equivalent to a ______________________________.

$$f(x) = b^x \cdot b^k = b^x b^k = ab^x \text{ where } a = b^k$$

Compare the graphs.

Original function	Horizontal translation	Vertical dilation
$f(x) = 2^x$ 	$g(x) = 2^{(x+3)}$ 	$g(x) = 8(2)^x$ 

Let $f(x) = 3^x$. Let $g(x)$ be a transformation of f. For each problem, name the transformation(s) of f.		
1. $g(x) = 9 \cdot 3^x$	2. $g(x) = \frac{3^x}{27}$	3. $g(x) = \frac{f(x)}{9}$

Recall: The **Power Property** for exponents: $(b^m)^n =$

$$(2^2)^3 = 2$$

Let's think of a **horizontal dilation** on an exponential function. Remember, horizontal means we are doing something the x-values. $f(x) = b^{(cx)}$, where b and c are constants.

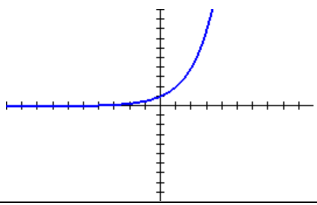
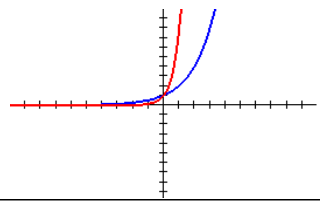
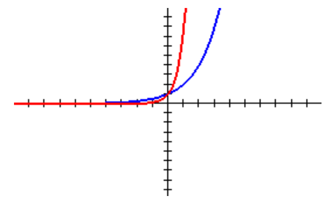
Write your questions and thoughts here!

Horizontal Dilation

Every horizontal dilation of an exponential function, $f(x) = b^{(cx)}$, is equivalent to a __________ __________________ of an exponential function.

$$f(x) = (b^c)^x \text{ where } b^c \text{ is a constant and } c \neq 0.$$

Compare the graphs.

Original function	Horizontal dilation	Change of base
$f(x) = 2^x$	$g(x) = 2^{(3x)}$	$g(x) = 8^x$
		

Let $f(x) = 3^x$. Let $g(x)$ be a transformation of f. For each problem, name the transformation(s) of f.

4. $g(x) = 9^x$

5. $g(x) = \left(f(x)\right)^{-3}$

The kth root of b

$b^{(1/k)}$ is the kth root of b.

$$b^{(1/k)} =$$

where k is a natural number and the root exists.

Evaluate the function at the given input values.

6. Let $h(x) = 2 \cdot 5^{x/2}$. Find $h(1)$

7. Let $h(x) = 6 \cdot 2^{x/3}$. Find $h(-2)$

2.4 Exponential Function Manipulation

AP Precalculus

2.4 Practice

Let $f(x)$ be a function on which a transformation occurs. Let $g(x)$ be a transformation of f. For each problem, name the transformation(s) of f.

1. $f(x) = 3^x$ and $g(x) = f(x) \cdot 27$

2. $f(x) = 9^x$ and $g(x) = 3^x$

3. $f(x) = 7^x$ and $g(x) = \frac{1}{7^x}$

4. $f(x) = 4^x$ and $g(x) = f(x) \cdot 16$

5. $f(x) = 4^x$ and $g(x) = \frac{f(x)}{16}$.

6. $f(x) = 5^x$ and $g(x) = \left(f(x)\right)^{-3}$

7. $f(x) = 6^x$ and $g(x) = \frac{f(x)}{36}$

8. $f(x) = 3^x$ and $g(x) = 27^x$

9. $f(x) = 4^x$ and $g(x) = -(16)^x$

10. $f(x) = 16^x$ and $g(x) = 2^x$

11. $f(x) = 7^x$ and $g(x) = -\frac{f(x)}{49}$

12. $f(x) = 2^x$ and $g(x) = \frac{f(x)}{16}$

13. $f(x) = 3^x$ and $g(x) = -3f(x)$

14. $f(x) = 2^x$ and $g(x) = f(x) \cdot 32$

Evaluate the function at the given input values.

15. Let $h(x) = 2 \cdot 3^{x/2}$. Find $h(1)$

16. Let $h(x) = 4 \cdot 4^{x/5}$. Find $h(2)$

17. Let $h(x) = 7 \cdot 2^{x/4}$. Find $h(-2)$

18. Let $h(x) = 2 \cdot 6^{x/3}$. Find $h(-1)$

2.4 Exponential Function Manipulation

2.4 Test Prep

19. **Calculator active.** A lake in the Cascade Mountains has frozen over during the winter. As spring brings warmer weather, the ice sheet begins to melt. The table below gives the area of the ice, is square feet, at various times, in days since the beginning of spring.

Time (days)	0	1	2
Area (millions of square feet)	2.5	2	1.6

The area of the ice can be modeled by the function $A(t) = ab^t$, where $A(t)$ is the area, in square feet, at time t days since the beginning of spring.

a. Use the given data to write two equations that can be used to find the values for constants a and b in the expression $A(t)$.

b. Find the values for a and b, then write the expression for $A(t)$.

c. Use the given data to find the average rate of change of the area from $t = 0$ to $t = 2$ days. Show the computations that lead to your answer.

d. Interpret the meaning of your answer from part c in the context of the problem.

e. Use the average rate of change found in part c to estimate the area of the ice, in square feet, at time $t = 3$ days. Show the work that leads to your answer.

f. Does the model found in Part b demonstrate exponential growth or exponential decay? Give a reason for your answer.