2.5.A Exponential Function Context and Data Modeling

Solutions

2.5.A Practice

AP Precalculus

Below is a table of values for exponential functions in the form $f(x) = a(b)^x + k$. Write the equation that represents each table.

1.

x	0	1	2	3	4
$f(x)$	2	10	34	106	322

8, 24, 72, 216

b → ·3, ·3, ·3

$y = a \cdot 3^x + k$

$2 = a \cdot 3^0 + k$

$2 = a + k$

$2 - a = k$

$10 = a \cdot 3^1 + k$

$10 = 3a + k$

$10 - 3a = k$

$2 - a = 10 - 3a$

$2a = 8$

$\boxed{a = 4}$

$2 - 4 = k$

$\boxed{-2 = k}$

$\boxed{f(x) = 4(3)^x - 2}$

2.

x	0	1	2	3	4
$f(x)$	8	13	23	43	83

5, 10, 20, 40

b → ·2, ·2, ·2

$y = a \cdot 2^x + k$

$8 = a \cdot 2^0 + k$

$8 = a + k$

$8 - a = k$

$13 = a \cdot 2^1 + k$

$13 - 2a = k$

$8 - a = 13 - 2a$

$\boxed{a = 5}$

$8 - 5 = k$

$\boxed{3 = k}$

$\boxed{f(x) = 5(2)^x + 3}$

3.

x	0	1	2	3	4
$f(x)$	5	29	149	749	3,749

24 120 600 3000

b → ·5 ·5 ·5

$y = a \cdot 5^x + k$

$5 = a \cdot 5^0 + k$

$5 - a = k$

$29 = a \cdot 5^1 + k$

$29 - 5a = k$

$5 - a = 29 - 5a$

$4a = 24$

$\boxed{a = 6}$

$5 - 6 = k$

$\boxed{-1 = k}$

$\boxed{f(x) = 6(5)^x - 1}$

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4. The table presents values of the function f for selected values of x.

x	-8	-7	-1	2
$f(x)$	30	40	158	280

a. Find an exponential regression $y = ab^x$ to model these data. Round to three decimals but store the original equation in your calculator.

$\boxed{y = 186.721(1.251)^x}$

b. Use the model stored in your calculator to predict the value of $f(0.5)$.

$\boxed{208.813}$

c. According to the model, when will $f(x) = 100$?

$\boxed{x = -2.792}$

d. According to the model, when will $f(x) = 1{,}000$?

$\boxed{x = 7.504}$

5. The table gives the number of cells c of a culture of bacteria found in a petri dish after t days.

Days (t)	3	5	10	13
Cells (c)	300	350	966	1,807

a. Use an exponential regression $c(t) = ab^t$ to model these data. Round to three decimals but store the original equation in your calculator.

$y = 155.504(1.204)^x$

b. According to the model, how many cells were in the culture of bacteria after 8 days? Use the model stored in your calculator, not the rounded answer from part a.

685.3485 cells

c. Use the model stored in your calculator to predict the number of bacteria cells after 31 days.

48,741.952 cells

d. How many days will it take for the culture to have 1 million cells?

$t = 47.295$
After 47 days (during the 48^{th} day).

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6. The table gives the population of a city, in millions, for selected years where $t = 2$ represents 2012 and $t = 5$ represents 2015.

Year	2012	2015	2018	2021
Population (millions)	0.801	0.933	1.36	1.85

a. Use an exponential regression $f(t) = ab^t$ to model these data. Round to three decimals but store the original equation in your calculator.

$y = 0.626(1.101)^x$

b. Use the model stored in your calculator to predict the population of the city in 2030.

4.295 million

c. During what year did the population reach 1.5 million?

$t = 9.072$. During 2019.

d. If the model holds, during what year will the population reach 3 million?

$t = 16.272$. During 2026.

7. Mr. Bean had a caffeinated soda at 8:00 a.m. The amount of caffeine (measured in mg) in Mr. Bean's system t hours is shown in the table below.

Hours (t)	1	3	8	12	13
Caffeine (mg)	128	85	47	20	18

a. Use an exponential regression $C(t) = ab^t$ to model these data. Round to three decimals but store the original equation in your calculator.

$$\boxed{y = 149.016(0.851)^x}$$

b. Use the model stored in your calculator to predict the amount of caffeine still in Mr. Bean's body after 15 hours.

$$\boxed{13.144 \text{ mg}}$$

c. How long will it take before the caffeine level is 5 mg?

$$\boxed{t = 20.971 \text{ hours}}$$

d. How long will it take before the caffeine level is 1 mg?

$$\boxed{t = 30.193 \text{ hours.}}$$

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2.5.A Test Prep

8. **Calculator active.** In the xy-plane, the graphs of the linear function L and the exponential function E both pass through the points $(0, 4)$ and $(1, 8)$. The exponential function E is in the form $E(x) = ab^x$. The function f is given by $f(x) = E(x) - L(x)$. What is the minimum value of f.

$L(x) = m(x - x_1) + y_1$

$m = \frac{8-4}{1-0} = 4$

$L(x) = 4(x - 0) + 4$

$L(x) = 4x + 4$

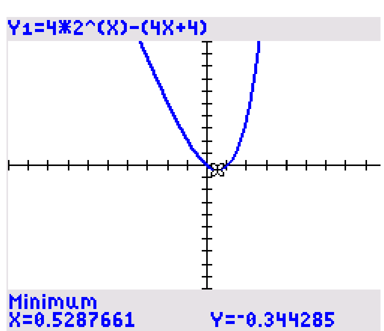
$E(x) = a \cdot b^x$

$b = 2 \leftarrow$ multiply 4 by 2 to get 8.

$a = 4 \leftarrow$ initial value

$E(x) = 4 \cdot 2^x$

$f(x) = E(x) - L(x)$



$\boxed{\begin{array}{c}\mathbf{-0.344} \\ \text{Remember, a value of f} \\ \text{is always a } y\text{-value.}\end{array}}$